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“The fear of the LORD is the
beginning of knowledge: but fools
despise wisdom and instruction.”

Proverbs 1:7

University of Alberta

Design and Analysis of Fast Spin Echo Imaging at 3.0 Tesla

By

Jason Kraig Mendes



A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment
of the requirements for the degree of **Master of Science**

in

Medical Physics

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Faculty of Graduate Studies and Research

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **Design and Analysis of Fast Spin Echo Imaging at 3.0 Tesla** submitted by **Jason Kraig Mendes** in partial fulfillment of the requirements for the degree of **Master of Science in Medical Physics**.



To my sister, thank you for
your love and support.

Abstract

Fast Spin Echo imaging, also known as Rapid Acquisition with Relaxation Enhancement or Turbo Spin Echo, is a routine part of clinical and research Magnetic Resonance Imaging. This thesis covers the design, implementation and evaluation of a Fast Spin Echo imaging sequence at 3.0 Tesla. It offers new insight into some of the fundamental relationships between magnetic field strength, Radio Frequency homogeneity, refocus pulse flip angles and the consequential image intensity variations. Fast spin echo imaging is shown to be a successful Black Blood Angiography technique and has been effectively applied to whole body imaging. Fast Spin Echo sequences can be successfully utilized at higher field strengths if barriers, such as increased power deposition and RF inhomogeneity, can be resolved.

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List of Abbreviations

2D	Two Dimensional
3D	Three Dimensional
BB	Black Blood
BW	Bandwidth
CP	Carr-Purcell
CPMG	Carr-Purcell-Meiboom-Gill
DBM	Doubly Balanced Mixers
EM	Electromagnetic
EMF	Electromotive Force
ETL	Echo Train Length
ETS	Echo Train Spacing
FID	Free Induction Decay
FOV	Field Of View
FS	Fat Suppression
FT	Fourier Transform
FSE	Fast Spin Echo
GE	Gradient Echo
MR	Magnetic Resonance
MRA	Magnetic Resonance Angiography
MRI	Magnetic Resonance Imaging
MT	Magnetization Transfer
NMR	Nuclear Magnetic Resonance
PD	Proton Density
PPM	Parts Per Million
RARE	Rapid Acquisition with Relaxation Enhancement

RF	Radio Frequency
ROI	Region Of Interest
SAR	Specific Absorption Ratio
SE	Spin Echo
SNR	Signal to Noise Ratio
TE	Echo Time
TH	Slice Thickness
TOF	Time Of Flight
TR	Repetition Time
TSE	Turbo Spin Echo

List of Symbols

σ	Conductivity
ρ	Density
Ψ	General wave equation
γ	Gyromagnetic ratio
ξ	Interaction energy
μ	Intrinsic magnetic moment, permeability
μ	Permeability
ε	Permittivity
$\hbar$	Plank's constant divided by 2π
θ	Radio frequency flip angle
κ	Wave number
λ	Wavelength
ω_0	Larmor frequency
μ_0	Permeability of free space
ε_0	Permittivity of free space
ω_{eff}	Effective angular frequency
ρ_f	Free charge
σ_f	Free surface charge
τ_g	Gradient pulse length
ϕ_g	Phase created by a gradient
Φ_M	Magnetic Flux
τ_p	Phase encode time
μ_r	Relative Permeability

ϵ_r	Relative permittivity
ω_{rf}	Angular frequency of the RF pulse
ϕ_{rf}	Phase of the RF pulse
τ_{rf}	Radio frequency pulse length
Δt_{read}	Sampling time of the readout signal
$ -\rangle$	Dirac's <i>bra-ket</i> notation for the “spin down” state
$ +\rangle$	Dirac's <i>bra-ket</i> notation for the “spin up” state
$\mathbf{B}_0$	Static magnetic field
$\mathbf{B}_1$	Radio frequency magnetic field
B_1^e	Pulse envelope modulation
$\mathbf{B}_{eff}$	Effective magnetic field
BW_{read}	Readout bandwidth
BW_{rf}	Radio frequency pulse bandwidth
$\mathcal{E}$	Electromotive force
E	Energy eigenvalue
E_z	Zeeman energy
FOV_p	Field of view in the phase direction
FOV_r	Field of view in the read direction
G_{pinc}	Phase encode gradient increment
G_r	Readout gradient strength
G_{ss}	Slice select gradient strength
G_x	Gradient strength in the x-direction
G_y	Gradient strength in the y-direction
G_z	Gradient strength in the z-direction
$\mathcal{H}$	Hamiltonian
$\hbar$	Plank's constant
i	Imaginary component ($\sqrt{-1}$)
$\mathbf{I}$	Spin angular momentum operator

$\hat{i}$	Unit vector along the x-axis
$\hat{j}$	Unit vector along the y-axis
J_f	Free current
k	Boltzmann constant
$\hat{k}$	Unit vector along the z-axis
K_f	Free current at a boundary
k_x	K-space x-coordinate
k_y	K-space y-coordinate
k_z	K-space z-coordinate
ℓ	Orbital quantum number
$\mathbf{M}$	Magnetization per unit volume
$\mathbf{M}_0$	Equilibrium magnetization per unit volume
m_s	Magnetic quantum number
M_{xy}	Transverse magnetization
$\hat{n}$	Normal unit vector
s	Spin quantum number
T	Temperature
T_1	Longitudinal relaxation time constant
T_2	Transverse relaxation time constant
T_2'	Nonreversible component of T_2
T_2^*	Reversible component of T_2
U	Energy density

1

Introduction

This thesis covers the design, implementation and evaluation of a Fast Spin Echo (FSE) imaging sequence on a 3.0 Tesla whole body Magnetic Resonance scanner. It offers new insight into some of the fundamental relationships between magnetic field strength, Radio Frequency (RF) homogeneity, RF flip angles and the consequential image intensity variations.

Chapter 1 presents a basic foundation of Nuclear Magnetic Resonance (NMR) to support the topics in this thesis. A fundamental quantum mechanical description of NMR is followed by a description of the vector model of NMR. This vector model is utilized to classically illustrate the basic procedures involved in a NMR experiment. The chapter concludes with how NMR is applied in tomographic imaging. A more exhaustive presentation of the basic physics and mathematics needed to understand NMR may be obtained through the many literature and internet resources available (1-9).

The 2nd chapter defines the spin echo and examines the various mechanisms involved in creating echoes. A discussion on the creation of stimulated echoes under non-ideal conditions is included. The concept of the spin echo is then applied to Magnetic Resonance Imaging (MRI). The shortcomings of spin echo imaging sequences in MRI are shown to foreshadow the development of FSE sequences. The chapter finishes with the basic building blocks and advantages of FSE sequences in MRI.

Chapter 3 covers the first goal of this thesis, which is the design of a FSE sequence suitable for imaging at 3.0 T. The design of the RF and magnetic gradient components of a FSE sequence are laid out. Difficulties implementing the sequence on the 3.0 Tesla MR scanner are discussed and examined experimentally. Some of the various contrast mechanisms of the designed FSE sequence are also experimentally demonstrated.

In chapter 4, the second goal of this thesis is achieved with the implementation of the FSE sequence to Magnetic Resonance Angiography (MRA). Characteristics of the FSE sequence make it ideal for the application of Black Blood Angiography. The FSE sequence is shown to successfully produce in-vivo black blood images at 3.0 T. A variety of sequence options are considered and examined experimentally to determine some optimum sequence configurations.

Chapter 5 covers the final goal of examining the viability of FSE imaging at 3.0 T. The need to reduce the refocus pulse flip angles to overcome RF heating problems when imaging with FSE at magnetic fields greater than 1.5 T is discussed. Reducing the refocus pulse flip angles is shown to lead to an unexpected increase in signal variation within the plane of the image. Chapter five brings new insight into this effect from a theoretically angle as well as experimentally at 1.5 T and 3.0 T.

1.1 Quantum Derivation of NMR

1.1.1 Intrinsic Magnetic Moment

In 1922 a refined experiment by Stern and Gerlach verified that a beam of neutral silver atoms passed through a non-uniform magnetic field would split into two components (10). However, the space quantization of orbital moments predicts an odd number $(2\ell + 1)$ of components, and silver atoms in their ground state have no orbital angular momentum ($\ell = 0$). Goudsmit and Uhlenbeck gave an explanation of this result in 1925, proposing that electrons must have a quantized, intrinsic magnetic moment in addition to the magnetic moment produced by orbital motion (11). This intrinsic magnetic moment μ is proportional to the spin angular momentum operator:

$$\mu = \gamma \hbar \mathbf{I} \quad [1.1]$$

The constant γ is a nuclei specific gyromagnetic ratio, $\hbar$ is Planck's constant divided by 2π and $\mathbf{I}$ is the spin angular momentum operator. The square of the spin angular momentum operator is given by its Cartesian components as:

$$\mathbf{I}^2 = I_x^2 + I_y^2 + I_z^2 \quad [1.2]$$

The components I_x , I_y and I_z of $\mathbf{I}$ are Hermitian operators that satisfy the commutation relations:

$$[I_x, I_y] = iI_z \quad [I_y, I_z] = iI_x \quad [I_z, I_x] = iI_y \quad [1.3]$$

The $\mathbf{I}^2$ relation of Eq. [1.2] and the commutation relations shown in Eq. [1.3] specify that the square of the spin angular momentum operator will commute with each of its components:

$$[\mathbf{I}^2, I_x] = [\mathbf{I}^2, I_y] = [\mathbf{I}^2, I_z] = 0 \quad [1.4]$$

The eigenvalues of $\mathbf{I}^2$ and any component of $\mathbf{I}$ can be simultaneously defined because $\mathbf{I}^2$ commutes with each of its components (12). Given the spin quantum number s , the eigenvalues of $\mathbf{I}^2$ are $s(s+1)$ and the eigenvalues of I_z have $(2s+1)$ allowed values identified by the magnetic quantum number m_s :

$$m_s \in [-s, -s+1, \dots, s] \quad [1.5]$$

An electron is not the only elementary particle with intrinsic spin and the spin quantum number is dependent on the type of particle being investigated. Estermann and Stern demonstrated that a proton, like the electron, also exhibits a quantized, intrinsic magnetic moment (13). A neutron, despite its zero overall charge, also exhibits an intrinsic magnetic moment that is opposite to its spin angular momentum (14). Bosons have integer values of spin ($s = 0, 1, 2, \dots$) while fermions have half-odd integer values of spin ($s = \frac{1}{2}, \frac{3}{2}, \dots$). Nuclei will possess an intrinsic magnetic moment if their constituents do not conspire to give zero spin. In this thesis, any elementary particle or nuclei that possess an intrinsic magnetic moment will be generally labeled as a “spin”.

1.1.2 Magnetic Moments in a Static Magnetic Field

A spin placed in a static magnetic field $\mathbf{B}_0$ will have a corresponding interaction energy ξ given by:

$$\xi = -\boldsymbol{\mu} \cdot \mathbf{B}_0 \quad [1.6]$$

The Cartesian reference frame is oriented so the static magnetic field lies along the z-axis:

$$\mathbf{B}_0 = [0, 0, B_0] \quad [1.7]$$

Using Eq. [1.1], [1.6] and [1.7], the Hamiltonian operator (interaction energy) of a magnetic moment in a static magnetic field can be defined in terms of the spin angular momentum operator:

$$\mathcal{H} = -\gamma \hbar B_0 I_z \quad [1.8]$$

The solution to the time independent Schrödinger wave equation requires a wave function describing the spin in addition to the Hamiltonian operator just derived.

1.1.3 Magnetic Moment of ^1H

Although there are several biologically important nuclei used in magnetic resonance, hydrogen nuclei are used most routinely due to their high gyromagnetic ratio and abundance in tissue (Table 1.1).

Table 1.1: List of Selected Nuclei

Nucleus	Spin	γ	Abundance ^a in the Human Body
Hydrogen ^1H	$\frac{1}{2}$	42.58 MHz/T	88M
Sodium ^{23}Na	$\frac{3}{2}$	11.27 MHz/T	80mM
Phosphorus ^{31}P	$\frac{1}{2}$	17.25 MHz/T	75mM
Oxygen ^{17}O	$\frac{5}{2}$	-5.77 MHz/T	16mM
Fluorine ^{19}F	$\frac{1}{2}$	40.08 MHz/T	4 μ M

^aRelative body abundance (1M = 1 molar = 1 mole/liter). Hydrogen molarity of water is 110M (5).

The ^1H nucleus consists of a single proton (fermion) and therefore has a spin quantum number of $\frac{1}{2}$. Much of the work presented in this thesis is applicable to other nuclei but only the hydrogen nuclei will be discussed.

1.1.4 Wave Function of ^1H in a Static Magnetic Field

The ^1H nucleus, or proton, has a spin of $\frac{1}{2}$ and from Eq. [1.5] the eigenvalues of I_z are:

$$m_s = \pm \frac{1}{2} \quad [1.9]$$

The two eigenstates of I_z are described using Dirac's *bra* and *ket* notation as $|+\rangle$ and $|-\rangle$ (12). These particular eigenstates do not have an analytical form, but have the property that:

$$I_z|+\rangle = \frac{1}{2}|+\rangle \quad I_z|-\rangle = -\frac{1}{2}|-\rangle \quad [1.10]$$

The state $|+\rangle$ is labeled “spin-up” as it corresponds to the protons with their spin aligned parallel to the external magnetic field. Analogously, the $|-\rangle$ state is labeled “spin-down” since these protons have their spin aligned anti-parallel to the external magnetic field. The proton spin is generally described by a wave function Ψ that is a superposition of the eigenstates $|+\rangle$ and $|-\rangle$ (15):

$$\Psi = c_+|+\rangle + c_-|-\rangle \quad [1.11]$$

The complex coefficients c_+ and c_- are defined such that $|c_+|^2$ and $|c_-|^2$ are the probabilities of finding the spin in the states $|+\rangle$ and $|-\rangle$ respectively. This wave function will be used in conjunction with the Hamiltonian to solve for the energy eigenvalues of a proton in a static magnetic field.

1.1.5 Energy Eigenvalues of ^1H in a Static Magnetic Field

Since I_z is the only operator in the Hamiltonian, the energy eigenstates of a proton in a static magnetic field correspond to the eigenstates of I_z . The energy eigenvalues are found by solving the time independent Schrödinger wave equation:

$$\mathcal{H}\Psi = E\Psi \quad [1.12]$$

The solution for a spin $\frac{1}{2}$ nucleus, such as the proton, has two energy eigenvalues:

$$E = \pm \frac{1}{2} \gamma \hbar B_0 \quad [1.13]$$

Figure 1.1 illustrates these two energy levels along with the associated eigenstates.

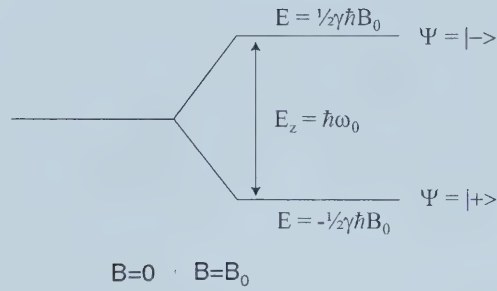


Figure 1.1: Energy level diagram for spin $\frac{1}{2}$ nuclei placed in a static magnetic field B_0 .

The energy difference between the two levels is denoted the Zeeman energy and has a value:

$$E_z = \gamma \hbar B_0 \quad [1.14]$$

A transition between the two levels will result in a spectral line at a characteristic frequency:

$$\omega_0 = \gamma B_0 \quad [1.15]$$

where ω_0 is known as the Larmor frequency. An ensemble of independent spins must be considered to determine the relative occupation probabilities of each eigenstate. For the ensemble, the thermal equilibrium distribution of probabilities is governed by Boltzmann statistics as:

$$\frac{\overline{|c_-|^2}}{\overline{|c_+|^2}} = e^{\frac{-E_z}{kT}} \quad [1.16]$$

where k is the Boltzmann constant, T is the temperature of the ensemble and E_z is the Zeeman energy (16).

Table 1.2: Ratio of N_- to N_+

B_0	Temperature		
	-100°C	25°C	100°C
1.5 T	0.9999972	0.9999984	0.9999987
3.0 T	0.9999944	0.9999967	0.9999974

The probabilities $|c_+|^2$ and $|c_-|^2$ are proportional to the populations, N_+ and N_- , of each respective energy level:

$$\frac{N_-}{N_+} = e^{\frac{-E_z}{kT}} \quad [1.17]$$

The ratios of N_- (number of “spin down” spins) to N_+ (number of “spin up” spins) for various temperatures and magnetic field strengths are given in Table 1.2.

1.2 Classical Description of NMR

1.2.1 Magnetization Vector Model

In a magnetic resonance (MR) experiment a spins reaction to external magnetic fields is observed through an interaction of the spin magnetic moment with an adjacent coil. Although the contribution of each spin magnetic moment is minute, the net magnetic moment produced by an ensemble of spins results in a significant signal being induced in the coil. The observed signal is therefore representative of the interaction of an ensemble of spins within a small volume or voxel rather than the interaction of each individual spin.

Defining a net magnetization vector per unit volume allows this representation of the spins to be made:

$$\mathbf{M} = \sum_{\text{volume}} \mu_i \quad [1.18]$$

The vector $\mathbf{M}$ is defined as the net magnetization per unit volume of spins. The theory of magnetic resonance can now be reasonably described in a classical sense for each voxel of spins rather than quantum mechanically for each individual spin. A more complete quantum mechanical derivation of the classical principles described in this thesis may be found in these references (1, 17-20).

1.2.2 Thermal Equilibrium Magnetization

Consider an ensemble of spins in the absence of any external magnetic fields. In thermal equilibrium the spins will arrange themselves in a random fashion such that the magnetic moments from all the spins will average to a zero net magnetization within a voxel.

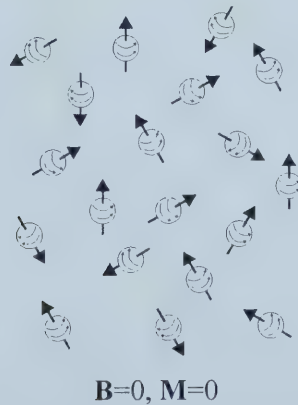


Figure 1.2: An ensemble of spins in the absence of an external magnetic field will orient themselves randomly. The resulting magnetization is zero.

If the same ensemble is placed in a static magnetic field $\mathbf{B}_0$, the spins will enter into a state that is either parallel or anti-parallel to the applied magnetic field. The thermal equilibrium magnetization of the voxel of spins will depend on the number of spins in each state. The relative population density of the two states in Eq. [1.17] depends on both the Zeeman energy and the temperature of the system. A thermal equilibrium magnetization per unit volume is defined as:

$$\mathbf{M}_0 = \mu(N_+ - N_-)\hat{\mathbf{k}} = \mu N_+ \left(1 - e^{\frac{-E_z}{kT}} \right) \hat{\mathbf{k}} \quad [1.19]$$

where μ is the nuclear moment of a single spin, N_+ is the number of spins per unit volume, k is the Boltzmann constant (not to be confused with the unit vector $\hat{\mathbf{k}}$ along the z-axis), T is the temperature of the ensemble and E_z is the Zeeman energy. Now that the thermal equilibrium magnetization has been defined, the question arises as to how a magnetization vector displaced from thermal equilibrium will behave in a static magnetic field.

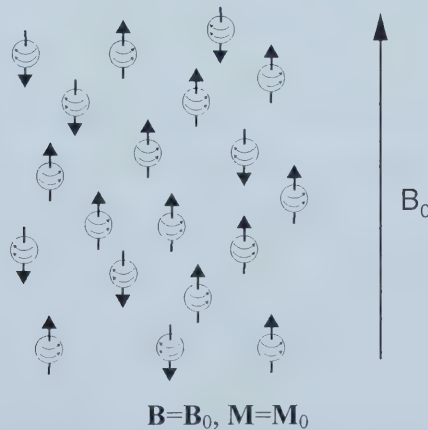


Figure 1.3: In the presence of an external magnetic field the magnetic moments will align themselves either parallel or anti-parallel to the external field.

1.2.3 Magnetization Vector in a Static Magnetic Field

A magnetic moment will experience a torque when exposed to an external magnetic field (21, 22). The torque placed on the magnetic moment defines the time rate of change of angular momentum. For an ensemble of spins:

$$\frac{d}{dt}(\sum \mathbf{I}\hbar) = \sum \boldsymbol{\mu} \times \mathbf{B} \quad [1.20]$$

This differential equation can be applied to the classical magnetization vector model:

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B} \quad [1.21]$$

The analytical solutions to the differential equation describing a magnetization vector in a static magnetic field $\mathbf{B}_0$ are:

$$M_z(t) = M_z(0) \quad [1.22a]$$

$$M_{xy}(t) = M_{xy}(0) \cdot e^{-i\omega_0 t} \quad [1.22b]$$

where ω_0 is the Larmor frequency, M_z is the longitudinal magnetization (along the applied magnetic field) and M_{xy} is the complex notation of the transverse magnetization (perpendicular to the applied magnetic field) defined as:

$$M_{xy} = M_x + iM_y \quad [1.23]$$

The result of Eq. [1.22] is that any longitudinal magnetization remains stationary while any transverse magnetization precesses in the perpendicular xy-plane at a frequency ω_0 (Fig 1.4).

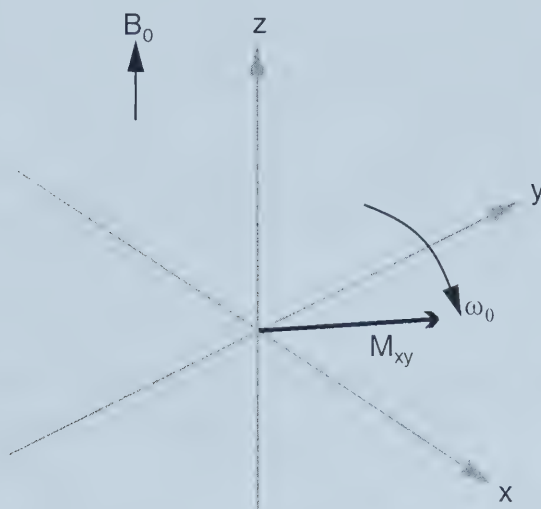


Figure 1.4: Under the influence of a static magnetic field, any transverse component of magnetization rotates in the xy-plane at the Larmor frequency. The direction of rotation is in the negative sense such that M_{xy} rotates from the positive y-axis toward the positive x-axis.

This behavior of a magnetization vector in a static magnetic field fails to describe how the magnetization vector returns to thermal equilibrium. The return to thermal equilibrium requires the mechanism of relaxation to be incorporated into the equations of motion.

1.2.4 Relaxation

Relaxation is a process that describes how a displaced magnetization vector returns to thermal equilibrium. In particular, longitudinal relaxation is how the longitudinal component of the magnetization vector returns to its thermal equilibrium value M_0 . The mechanism behind longitudinal relaxation is an energy exchange between resonant nuclei and the surrounding molecular lattice (1, 23, 24).

Bloch and Purcell demonstrated that the longitudinal relaxation rate is related to the degree that M_z has been removed from thermal equilibrium (25, 26):

$$\frac{dM_z}{dt} = \frac{M_0 - M_z(t)}{T_1} \quad [1.24]$$

The spin-lattice relaxation time T_1 is dependent on both the sample and the static magnetic field strength (27). Relaxation times for various tissues at different field strengths are given in Table 1.3. The solution to Eq. [1.24] is:

$$M_z(t) = M_0 + [M_z(0) - M_0]e^{\frac{-t}{T_1}} \quad [1.25]$$

In addition to the longitudinal component returning to M_0 , thermal equilibrium also requires there to be no transverse magnetization. The decay of transverse magnetization to zero is known as transverse relaxation and involves both reversible and non-reversible processes. The relaxation of transverse magnetization is governed by the following differential equation:

$$\frac{dM_{xy}}{dt} = \frac{-M_{xy}(t)}{T_2^*} \quad [1.26]$$

where T_2^* is the sample specific transverse relaxation time including both reversible and non-reversible contributions. The non-reversible contributions arise from mechanisms such as spin-spin interactions and spectrum broadening, while the reversible contributions arise from magnetic field inhomogeneities (2).

Table 1.3: Approximate Relaxation Times of Selected Tissues

Tissue	T ₁ (ms)		T ₂ (ms)	
	1.5 Tesla	3.0 Tesla	1.5 Tesla	3.0 Tesla
Gray Matter ^b	1000	1450	100	110
White Matter ^b	550	800	80	80
CSF ^b	4250	4150	2200	2200
Liver ^a	600	800	50	40
Spleen ^a	1050	1400	80	60
Pancreas ^a	600	750	50	50
Muscle ^a	850	900	30	40
Fat ^a	350	400	60	70
Prostate ^a	1350	1600	90	80

^aRelaxation times from (28).

^bRelaxation times from (5, 29).

The total transverse relaxation time can be expressed as a combination of two time constants:

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'} \quad [1.27]$$

where T_2 is the spin-spin time constant representing non-reversible processes and T_2' describes the reversible processes. The solution to Eq. [1.26] describing the return of transverse magnetization to equilibrium is:

$$M_{xy}(t) = M_{xy}(0) \cdot e^{\frac{-t}{T_2^*}} \quad [1.28]$$

Figure 1.5 illustrates the return to thermal equilibrium for both fat and muscle samples at a field strength of 3.0 Tesla after the magnetization has been fully excited into the transverse plane.

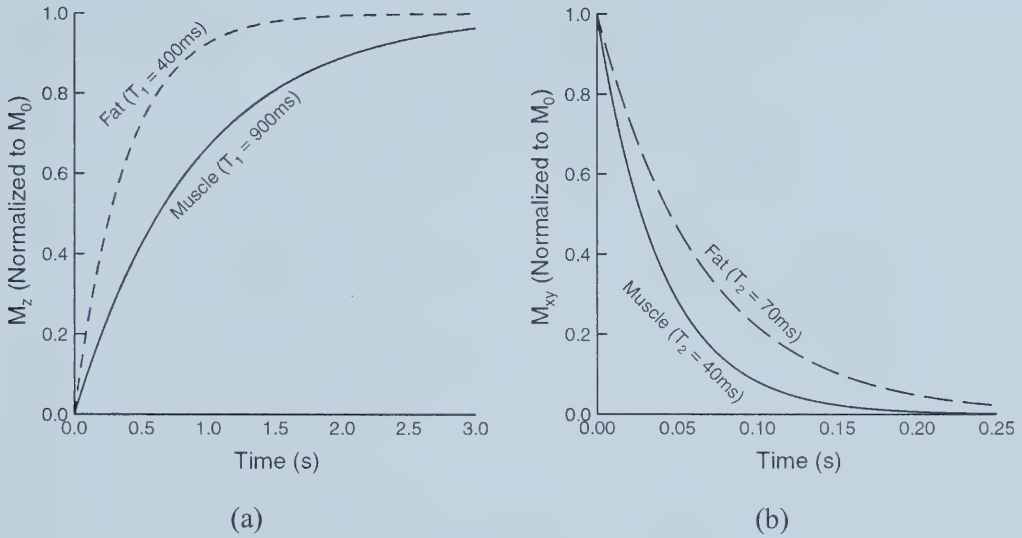


Figure 1.5: A magnetization vector excited into the transverse plane will return to thermal equilibrium through (a) longitudinal relaxation and (b) transverse relaxation. The transverse relaxation time in (b) does not include any reversible components.

In describing the relaxation of a magnetization vector back to thermal equilibrium, no mention has been made as to how or why the magnetization was disturbed from its equilibrium state to begin with. To understand why a displacement of the magnetization vector is necessary in a NMR experiment, the process of observing the magnetization vector must first be discussed.

1.2.5 Observing Magnetization

Faraday's law of induction describes the electromotive force (*emf*) that is induced in a coil due to a magnetic flux experienced by the coil (21, 30):

$$\mathcal{E} = \frac{-d\Phi_M}{dt} \quad [1.29]$$

where $\mathcal{E}$ is the *emf* and Φ_M is the flux through the coil given by:

$$\Phi_M = \int \mathbf{B} \cdot d\mathbf{A} \quad [1.30]$$

$d\mathbf{A}$ is a differential element of surface area and the integration is carried out over the entire surface. If a stationary coil is placed adjacent to a voxel of spins in a static magnetic field, depending on the orientation of the coil, the static magnetic field will create a magnetic flux through the coil. However, since this flux does not vary with time, there will be no induced *emf* in the coil. Similarly, the longitudinal magnetization M_z of the spins may produce a magnetic flux through the coil but will not induce an *emf* since the longitudinal magnetization component has no variation with time (Eq. [1.22a]). The result is not the same when the transverse component of the magnetization is considered. As shown in Eq. [1.22b], any transverse magnetization will rotate in the transverse plane at the Larmor frequency. This time dependent magnetic field will induce a sinusoidal *emf* in an adjacent coil at the Larmor frequency (6).

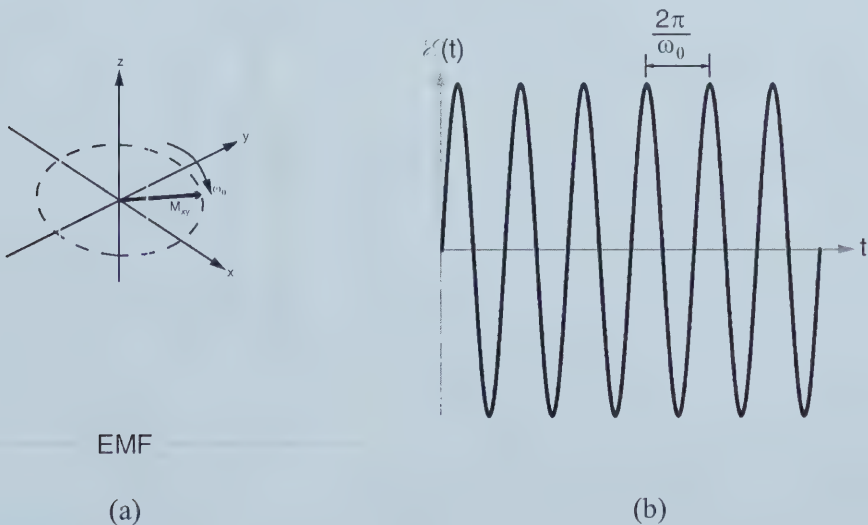


Figure 1.6: The time dependent magnetic field caused by the rotation of M_{xy} will induce an electromotive force in a suitably aligned coil (a). The *emf* will be a sinusoidal signal at the Larmor frequency (b).

The *emf* induced will be maximized if $d\mathbf{A}$ is aligned so that it resides in the transverse plane (i.e. the central axis of the coil is perpendicular to the static magnetic field). The *emf* signal in the coil is called a free induction decay or FID signal and allows the transverse magnetization to be observed.

In a real NMR experiment the magnetization vectors from several nearby voxels will contribute to the detected *emf* and the FID will never be a perfect sinusoidal signal. For example, the static magnetic field of a scanner will not be perfectly uniform over the entire region of interest (ROI). This will result in the magnetization vector of each voxel having a slightly different angular frequency since ω_0 is dependent on the static magnetic field. Figure 1.7 depicts three voxels in a non-uniform static magnetic field and the resulting FID signal. A mathematical tool called the Fourier transform is utilized to help make sense of the intricate FID signals acquired.

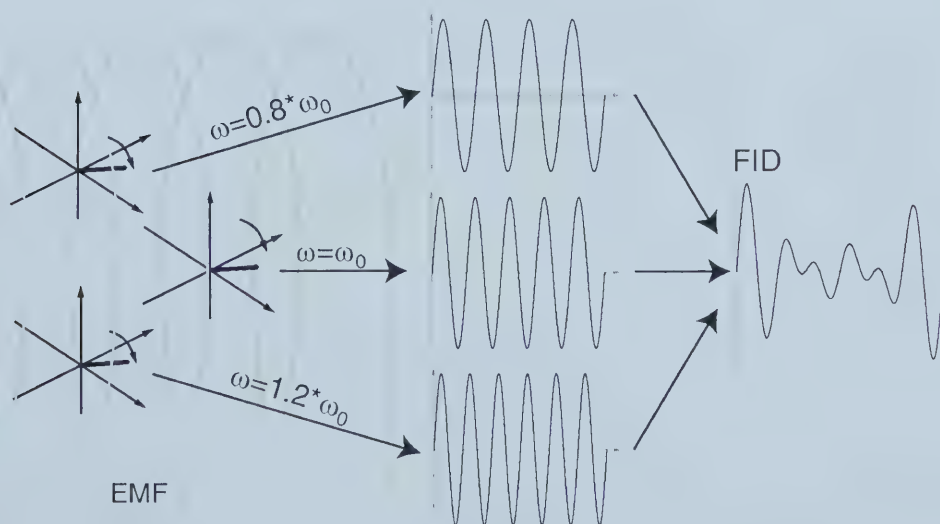


Figure 1.7: The resulting FID signal from three voxels experiencing a non-uniform static magnetic field is shown.

1.2.6 The Fourier Transform

The Fourier transform is an important mathematical tool in NMR that provides information about the temporal or spatial spectral content of a signal. It is not convenient to present a detailed description of the Fourier transform in this thesis. Instead, some important definitions and relations will be given and more detailed information may be obtained through these references (31-34). The Fourier and inverse Fourier transforms are defined as:

$$S(\omega) = F\{s(t)\} = \int_{-\infty}^{\infty} s(t) e^{-i\omega t} dt \quad [1.31a]$$

$$s(t) = F^{-1}\{S(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{i\omega t} d\omega \quad [1.31b]$$

where $s(t)$ is the signal of interest and $S(\omega)$ is the Fourier transform of the signal. The notation $F\{\}$ and $F^{-1}\{\}$ will be used to denote the Fourier transform and the inverse Fourier transform respectively. Table 1.4 contains a number of useful properties of the Fourier transform while some common signals used in NMR and the corresponding Fourier transforms are presented in Table 1.5.

Table 1.4: Selected Properties of the Fourier Transform

$s(t)$	$S(\omega)$
$s(a \cdot t)$	$\frac{1}{ a } S\left(\frac{\omega}{a}\right)$
$s(t) * h(t)$	$S(\omega) \cdot H(\omega)$
$s(t) \cdot h(t)$	$\frac{1}{2\pi} S(\omega) * H(\omega)$

Table 1.5: Selected Fourier Transform Pairs

$s(t)$	$S(\omega)$
1	$2\pi\delta(\omega)$
$\delta(t)$	1
$\delta(t - t_0)$	$e^{-i\omega t_0}$
$e^{i\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\text{rect}\left(\frac{t}{\tau}\right)$	$\tau \cdot \text{sinc}\left(\frac{\omega\tau}{2\pi}\right)$
$\frac{\omega_0}{\pi} \text{sinc}\left(\frac{\omega_0 t}{\pi}\right)$	$\text{rect}\left(\frac{\omega}{2\omega_0}\right)$
$\cos(\omega_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin(\omega_0 t)$	$-i\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$

If the Fourier transform is applied to the FID signal obtained in the example of Fig. 1.7, the three spectral constituents can be observed.

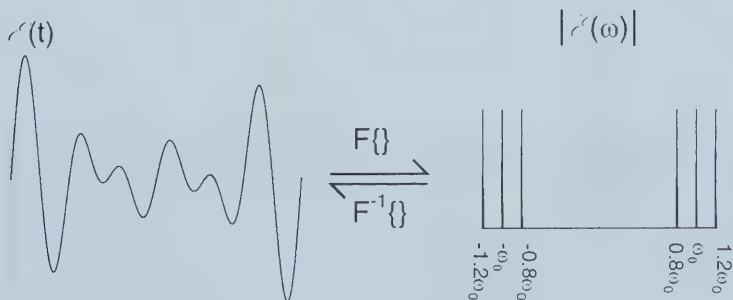


Figure 1.8: Fourier transform of the FID signal derived in Fig. 1.7 from three voxels experiencing a non-uniform magnetic field. The Fourier transform contains three spectral components at positive frequencies as well as three identical components at negative frequencies.

For all of the *emf* signals derived so far, the Fourier transform will always contain a mirror of the spectral information since the transform is unable to distinguish between positive and negative frequencies. This problem lies not in the Fourier transform itself but with the loss of phase information when detecting the *emf* signal. Both phase and magnitude information for the *emf* signal should be obtained to allow the Fourier transform to distinguish between positive and negative frequencies. This can be accomplished with the use of a quadrature detector.

1.2.7 Quadrature Signal Detection

A quadrature detector is a device that is able to extract both phase and magnitude information from the *emf* signal induced in the coil. A brief block diagram of a quadrature detector is given in Fig. 1.9. The detector typically contains a set of doubly balanced mixers (DBM), filters, and amplifiers with a 90° phase shifter. The *emf* signal and a reference signal are input into the detector and the real and imaginary components of the *emf* signal are output. The reference signal will depend on both the scanner hardware and the NMR experiment itself. For a detailed description of how the detector works, refer to the following (4, 6).

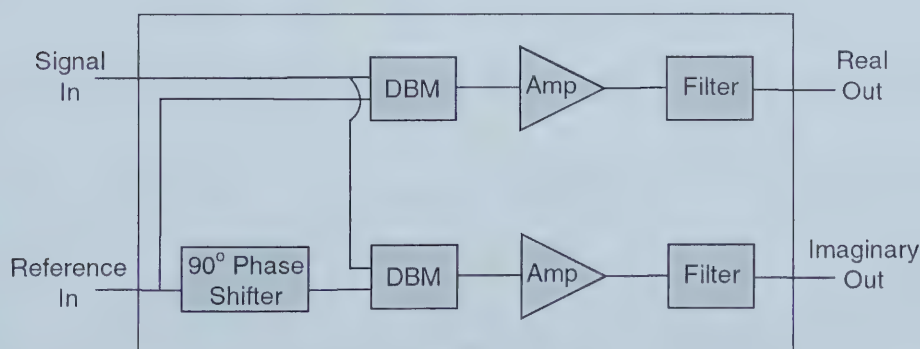


Figure 1.9: Both magnitude and phase information may be obtained from an *emf* signal with the use of a quadrature detector shown.

In a NMR experiment the actual precession frequency of each voxel of spins is less vital than the relation of this angular frequency to other voxels. If the Larmor frequency is subtracted off each of the angular frequencies involved in the experiment, the relative frequencies of each voxel are still preserved. In the frequency domain this is analogous to shifting the entire spectrum by a frequency ω_0 . In the time domain this is equivalent to demodulating the signal and can be accomplished by supplying a suitable reference signal to the quadrature detector. When describing this process mathematically, the concept of the rotating reference frame is utilized. The rotating frame allows an easier mathematical and visual representation of the magnetization vectors while still preserving information about how the magnetization vectors from one voxel to the next are related.

1.2.8 Rotating Reference Frame

All equations previous to this section have been derived in a stationary coordinate system that will be labeled the laboratory reference frame. The rotating reference frame differs from the laboratory reference frame in its rotation about the z-axis at a constant angular frequency. To differentiate the two reference frames, x' , y' and z' will be used to denote three orthogonal axes of the rotating frame as opposed to x , y and z for the laboratory frame.

The mathematical relations between the unit vectors of the two reference frames are:

$$\hat{\mathbf{i}}' = \cos(\omega t)\hat{\mathbf{i}} - \sin(\omega t)\hat{\mathbf{j}} \quad [1.32a]$$

$$\hat{\mathbf{j}}' = \sin(\omega t)\hat{\mathbf{i}} + \cos(\omega t)\hat{\mathbf{j}} \quad [1.32b]$$

$$\hat{\mathbf{k}}' = \hat{\mathbf{k}} \quad [1.32c]$$

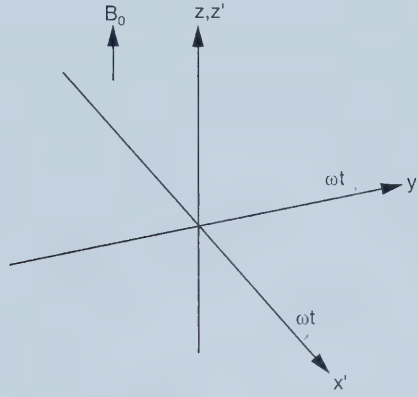


Figure 1.10: The rotating reference frame rotates about the z -axis at a constant angular frequency.

The rotating reference frame is an important tool in describing many NMR processes (35). In particular, it will allow a simplified description of how a magnetization vector behaves under the influence of more complex magnetic fields.

1.3 Radio Frequency Magnetic Fields

1.3.1 RF Waves in a Non-conducting Tissue

Maxwell's equations inside a region that is void of any free charge or free current are:

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 0 & \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \nabla \times \mathbf{B} &= \mu \epsilon \frac{\partial \mathbf{E}}{\partial t} \end{aligned} \quad [1.33]$$

where ϵ is the permittivity of the tissue (36), μ is the permeability of the tissue (37-40), $\mathbf{E}$ is the electric field and $\mathbf{B}$ is the magnetic field (21, 22, 30).

Table 1.6: Relative^a tissue permittivity

Tissue Type	64 MHz ϵ_r	128 MHz ϵ_r	171 MHz ϵ_r	256 MHz ϵ_r
CSF	97	84	79	74
Air	1	1	1	1
Muscle	72	64	62	60
Bone	17	15	14	14
Skin	77	62	57	53
Lens	50	43	41	39
Brain	83	63	58	53
Cartilage	63	53	50	48

Values obtained from reference (41).

$$^a \epsilon_r = \frac{\epsilon}{\epsilon_0}$$

The permittivity and permeability of a tissue are both frequency and temperature dependent. Table 1.6 demonstrates the frequency dependence of permittivity for several tissues at room temperature. For a monochromatic electromagnetic field linearly polarized along the z-direction, Eq. [1.33] has real solutions of:

$$\mathbf{E}(x, t) = -E_1 \cos(\kappa x - \omega t + \phi) \hat{\mathbf{k}} \quad [1.34a]$$

$$\mathbf{B}(x, t) = B_1 \cos(\kappa x - \omega t + \phi) \hat{\mathbf{j}} \quad [1.34b]$$

where κ is the propagation or wave number, ω is the angular frequency of the RF pulse, ϕ is the initial phase of the RF pulse, x is the position along the non-conducting tissue, B_1 is the strength of the magnetic field and E_1 is the strength of the electric field calculated as:

$$E_1 = \frac{B_1}{\sqrt{\epsilon \mu}} \quad [1.35]$$

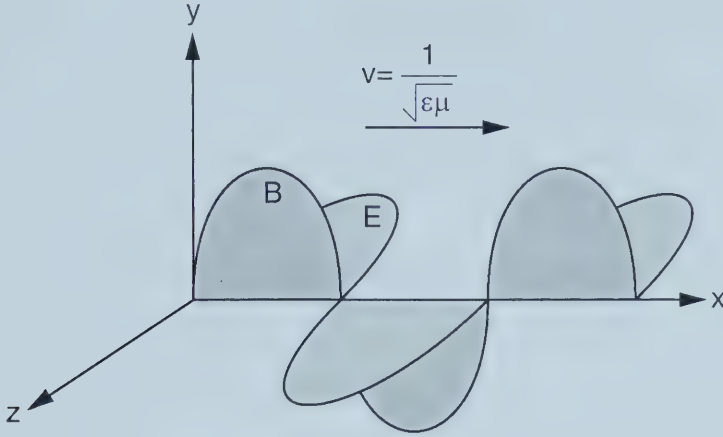


Figure 1.11: The transverse wave described by Eq. [1.34] has the magnetic field polarized along the y-axis, an electric field polarized along the z-axis and propagates along the x-axis at a velocity determined by the permittivity and permeability of the sample.

Figure 1.11 illustrates a linearly polarized, monochromatic plane wave propagating in a non-conducting media. The electric and magnetic fields are in phase and are not attenuated as the wave propagates through the media. The wavelength of the electromagnetic wave is given as:

$$\lambda = \frac{2\pi}{\omega\sqrt{\epsilon\mu}} \quad [1.36]$$

The energy density (energy per unit volume stored in an electromagnetic wave) is defined as:

$$U = \frac{1}{2} \left(\epsilon E^2 + \frac{1}{\mu} B^2 \right) \quad [1.37]$$

For a non-conducting tissue the electric and magnetic field contributions are the same and the energy density is:

$$U = \epsilon E_1^2 \cos^2(\mathbf{k}\mathbf{x} - \omega t + \phi) \quad [1.38]$$

Since the average of $\cos^2$ over a complete cycle is $\frac{1}{2}$, the average energy density over a complete cycle is:

$$\langle U \rangle = \frac{1}{2} \epsilon E_1^2 \quad [1.39]$$

Notice that the average energy density is unchanged with penetration depth into the media. Although there is no energy lost due to the non-conducting tissue itself, the electromagnetic wave will still be dissipated in other processes such as spin excitation.

1.3.2 RF Waves in a Conducting Tissue

In a vacuum or a dielectric material such as glass or pure water it is reasonable to set the free charge density (ρ_f) and free current density ($\mathbf{J}_f$) to zero. However, in a conducting media where charge is allowed to flow, the free charge density and free current density cannot be generally assumed to be zero. Without this assumption on ρ_f and $\mathbf{J}_f$, Maxwell's equations become:

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{1}{\epsilon} \rho_f & \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \nabla \times \mathbf{B} &= \mu \sigma \mathbf{E} + \mu \epsilon \frac{\partial \mathbf{E}}{\partial t} \end{aligned} \quad [1.40]$$

where σ is the conductivity of the conductor (Table 1.7).

Table 1.7: Conductivity of selected tissues

Tissue Type	64 MHz σ (S/m)	128 MHz σ (S/m)	171 MHz σ (S/m)	256 MHz σ (S/m)
CSF	2.07	2.14	2.17	2.21
Air	0.00	0.00	0.00	0.00
Muscle	0.71	0.74	0.76	0.78
Bone	0.06	0.07	0.07	0.08
Skin	0.49	0.54	0.57	0.61
Lens	0.29	0.31	0.32	0.34
Brain	0.40	0.46	0.49	0.53
Cartilage	0.45	0.49	0.51	0.54

Values obtained from reference (36).

For a conducting media, any free charge ρ_f will necessarily dissipate in a characteristic time τ , depending on how good the conductor is. The characteristic time is defined as:

$$\tau = \frac{\epsilon}{\sigma} \quad [1.41]$$

A good conductor has a characteristic time that is much less than the other relevant times in the analysis ($\tau \ll 1/\omega$). Since the dissipation of charge is a transient behavior, the charge will first be allowed to dissipate and from then on $\rho_f = 0$. Maxwell's equations given in Eq. [1.40] will then have real solutions of:

$$\mathbf{E}(x, t) = -E_1 e^{-\kappa_+ x} \cos(\kappa_+ x - \omega t + \phi) \hat{\mathbf{k}} \quad [1.42a]$$

$$\mathbf{B}(x, t) = B_1 e^{-\kappa_+ x} \cos(\kappa_+ x - \omega t + \phi + \phi_\kappa) \hat{\mathbf{j}} \quad [1.42b]$$

where $\kappa = \kappa_+ + i\kappa_-$ is the complex wave number with a modulus of $|\kappa|$ and a phase of ϕ_κ .

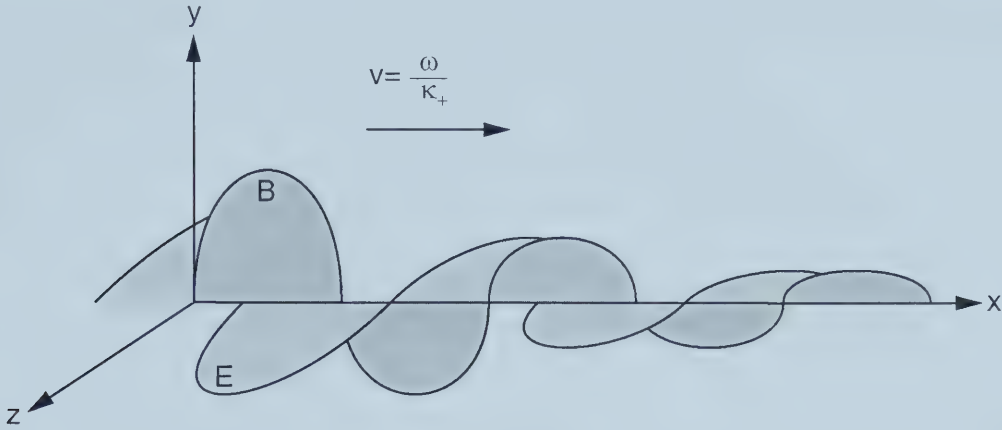


Figure 1.12: A conducting media attenuates a transverse wave propagating along the x -axis. The magnetic and electric field components are not necessarily in phase inside a conducting media.

The components of the wave number are defined as:

$$\kappa_{\pm} = \omega \sqrt{\frac{\epsilon\mu}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega} \right)^2} \pm 1 \right]^{\frac{1}{2}} \quad [1.43]$$

The electric field strength is related to the magnetic field strength through the wave frequency and the modulus of the wave number as:

$$E_1 = \frac{\omega}{|\kappa|} B_1 \quad [1.44]$$

A few important observations should be made about the propagation of the electromagnetic wave. First, the magnetic and electric fields are not necessarily in phase as they propagate through a conducting media. Second, the magnetic and electric fields are attenuated as the wave penetrates into a conducting media.

The skin depth is the distance the wave penetrates into the media before having its amplitude reduced by $1/e$.

$$d = \frac{1}{\kappa_-} \quad [1.45]$$

The wavelength is calculated from the real component of the wave number as:

$$\lambda = \frac{2\pi}{\kappa_+} \quad [1.46]$$

Unlike the non-conducting media, the magnetic and electric fields do not contribute to the energy density equally. The average energy density in a conducting media is:

$$\langle U \rangle = \frac{1}{4} \epsilon E_1^2 e^{-2\kappa_- x} \left[1 + \sqrt{1 + \left(\frac{\sigma}{\epsilon \omega} \right)^2} \right] \quad [1.47]$$

The energy density decreases as the electromagnetic wave propagates into a conducting media. In addition, it is the magnetic contribution (second term) that always dominates as opposed to the non-conducting media when the contributions from the electric and magnetic fields were equal. The average power per unit volume delivered to a conducting media by an electromagnetic wave is:

$$\langle P \rangle = \frac{1}{2} \sigma E_1^2 e^{-2\kappa_- x} \quad [1.48]$$

The power lost due to RF penetration is heavily influenced by the conductivity of the sample.

1.3.3 RF Probes

An ideal RF coil should be able to transmit a transverse, circularly polarized magnetic field that is uniform in both amplitude and phase across the functional volume of the coil (42, 43). However, in reality the magnetic field produced by a coil is often linearly or elliptically polarized (44). Such a magnetic field can be decomposed into two counter-rotating circularly polarized magnetic fields. The minor counter-rotating component does not couple to the nuclear spins and will contribute to signal losses and noise. The major component will couple with spins and provide excitation and reception. The electric field does not play a useful role in NMR processes but will contribute to power deposition in the sample and increased noise on reception. The reciprocity relation states that a coil well designed to excite spins at a specific location is also well designed to receive a signal from the same spins (45, 46). Transmission and reception in NMR experiments are accomplished with surface or volume coils or a combination of the two. Only a brief description of each coil type will be presented in this section and a more detailed examination is found in the following references (41, 46-49)

The simplest surface coils are single loops of wire that are placed directly over the region of interest in a sample (Fig. 1.13). The sensitivity of a surface coil falls off quickly along the axial direction and the coil is only sensitive for a fraction of the loop diameter (50). If the region of interest lies near the surface of the sample then a surface coil will provide a SNR and sensitivity advantage over a volume coil (51, 52).

A single surface coil operates in a linear rather than a quadrature mode. The minor counter-rotating component of the linear field does not couple with the spins and this results in a $1/\sqrt{2}$ loss in SNR. The SNR loss can be regained if two surface coils are aligned to operate in a quadrature mode. Surface coils are especially popular for imaging structures that are near the surface of the sample. For deep lying structures or when large regions of interest are needed, a volume coil is a more appropriate selection.

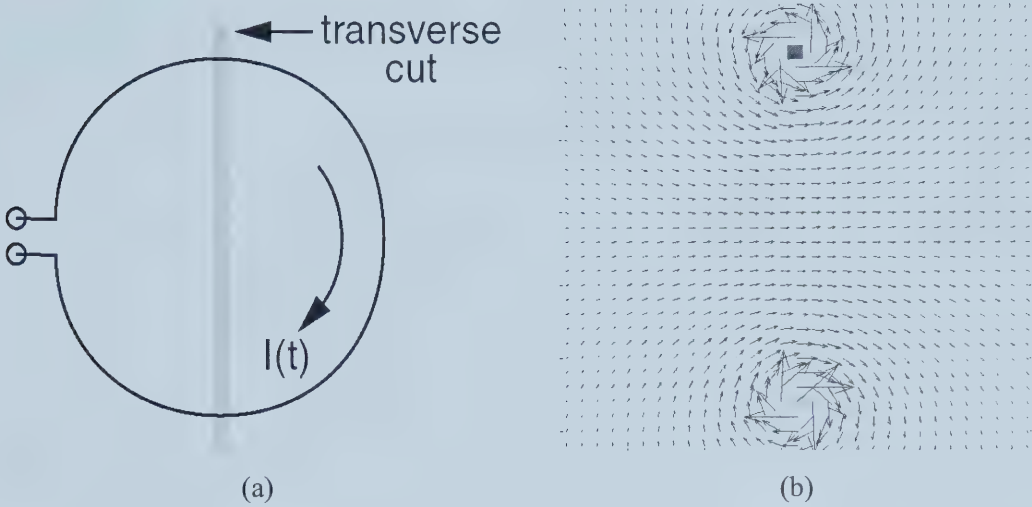


Figure 1.13: The single loop surface coil in (a) produces the magnetic field distribution shown in the transverse cut through the coil (b).

Although there are many popular volume coil designs, the birdcage is most popular configuration used in NMR (53-58). A birdcage coil consists of a series of identical wire loops connected along the surface of a cylinder.

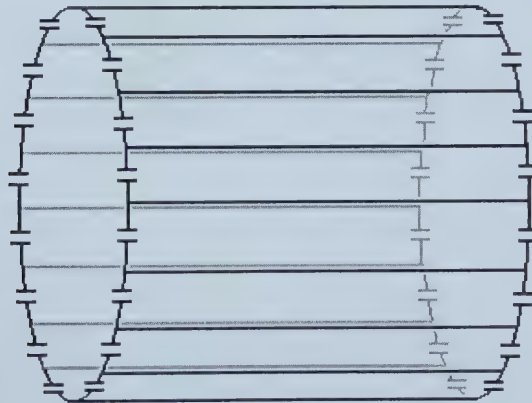


Figure 1.14: Example of one variation of a birdcage volume coil.

If the current in each leg of the coil is:

$$I(t) = I_0 \sin(\omega_{\text{RF}} t + \phi) \quad [1.49]$$

then the coil will operate in a quadrature mode and provide excellent radial field homogeneity over the functional volume of the coil (Fig 1.15).

Sometimes a volume coil is used to transmit in a NMR experiment and a surface coil is used for reception. In this way the excellent field homogeneity of a volume coil and the increased sensitivity of a surface coil may be exploited.

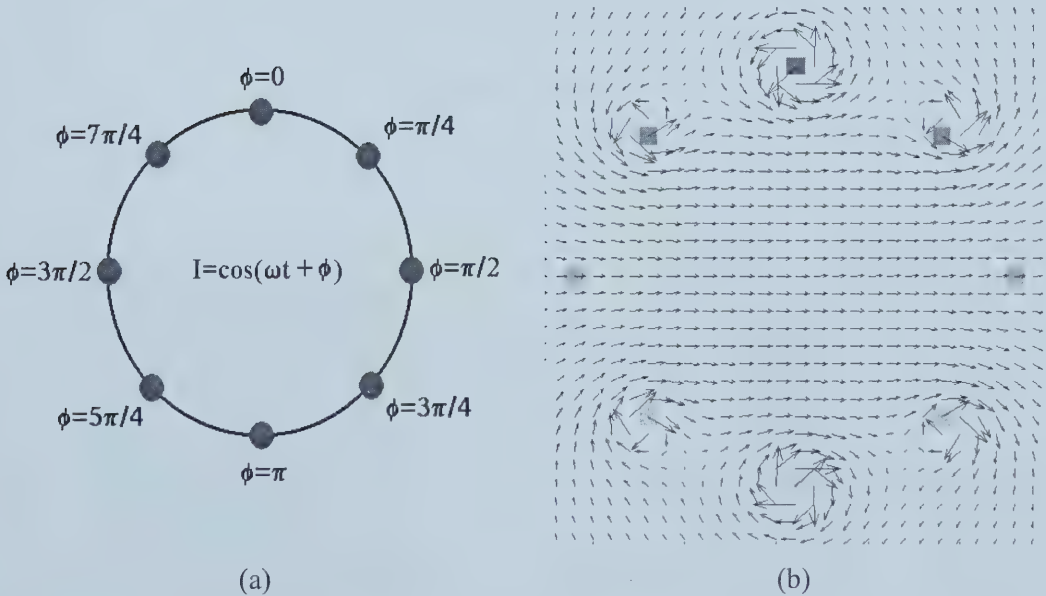


Figure 1.15: The transverse cut (a) through a birdcage coil shows the phase offset of the current in each leg of an 8-element coil. The same transverse cut in (b) shows the resulting magnetic field distribution resulting from the current distribution shown in (a).

1.3.4 Radio Frequency Pulses

A circularly polarized radiofrequency (RF) field will be applied to the system in addition to the static magnetic field $\mathbf{B}_0$. This RF field is described as:

$$\mathbf{B}_1(t) = B_1^e(t) \cdot [\cos(\omega_{rf}t + \phi_{rf}), -\sin(\omega_{rf}t + \phi_{rf}), 0] \quad [1.50]$$

where ϕ_{rf} is the initial phase of the RF pulse and $B_1^e(t)$ is the RF pulse envelope function. Two examples of RF pulse envelopes have been shown in Fig. 1.16. The RF pulse equation presented is not the only RF field that may be used to manipulate spins in a NMR experiment. However, for this thesis all NMR experiments will use RF pulses described by Eq. [1.50]. The total magnetic field experienced by a voxel of spins is now:

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1(t) \quad [1.51]$$

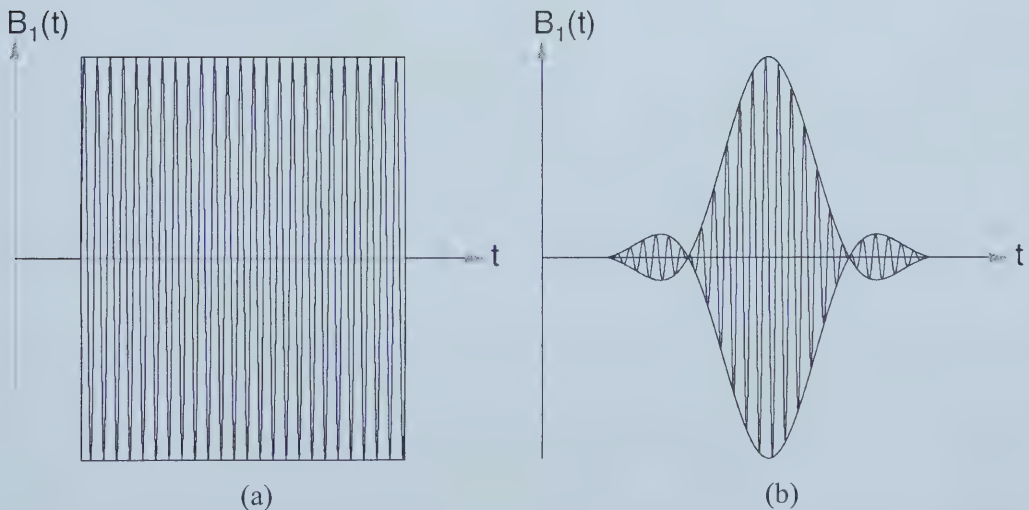


Figure 1.16: A square envelope (a) and a sinc envelope (b) are just two of many envelope functions used to modulate RF pulses in NMR experiments.

Using Eq. [1.21], the differential equations of motion describing the magnetization vector in the laboratory frame are:

$$\frac{dM_x}{dt} = \gamma [M_y B_0 + M_z B_1^e(t) \sin(\omega_{rf} t + \phi_{rf})] \quad [1.52a]$$

$$\frac{dM_y}{dt} = \gamma [M_z B_1^e(t) \cos(\omega_{rf} t + \phi_{rf}) - M_x B_0] \quad [1.52b]$$

$$\frac{dM_z}{dt} = -\gamma B_1^e(t) [M_x \sin(\omega_{rf} t + \phi_{rf}) + M_y \cos(\omega_{rf} t + \phi_{rf})] \quad [1.52c]$$

A closed loop solution to Eq. [1.52] is not available for an arbitrary envelope function. The concepts of the rotating frame ($\omega = \omega_{rf}$) will be utilized to reduce the complexity of the equations of motion. For the rotating reference frame Eq. [1.21] becomes:

$$\frac{d\mathbf{M}'}{dt} = \gamma \mathbf{M}' \times \left(\mathbf{B}' + \frac{\boldsymbol{\omega}_{rf}}{\gamma} \right) \quad [1.53]$$

where the vector $\boldsymbol{\omega}$ is defined as:

$$\boldsymbol{\omega}_{rf} = -\omega_{rf} \hat{\mathbf{k}}' \quad [1.54]$$

The equations of motion describing the magnetization vector in the rotating reference frame are then:

$$\frac{dM_{x'}}{dt} = \Delta\omega M_{y'} + \gamma B_1^e(t) \sin(\phi_{rf}) M_{z'} \quad [1.55a]$$

$$\frac{dM_{y'}}{dt} = -\Delta\omega M_{x'} + \gamma B_1^e(t) \cos(\phi_{rf}) M_{z'} \quad [1.55b]$$

$$\frac{dM_{z'}}{dt} = -\gamma B_1^e(t) [\sin(\phi_{rf}) M_{x'} + \cos(\phi_{rf}) M_{y'}] \quad [1.55c]$$

The angular frequency difference $\Delta\omega$ is defined as:

$$\Delta\omega = \omega_0 - \omega_{\text{rf}} \quad [1.56]$$

Unfortunately there is still no closed loop solution to Eq. [1.55] for an arbitrary RF pulse envelope. The benefit of using the rotating frame is that the equations of motion are less complex than in the laboratory frame. To illustrate this point, first consider the trajectory of the magnetization vector in the absence of an applied RF field. The laboratory frame equations reduce to the solutions already derived in Eq. [1.22]. Conversely, the differential equations of motion in the rotating frame are:

$$\frac{dM_{x'}}{dt} = \frac{dM_{y'}}{dt} = \frac{dM_{z'}}{dt} = 0 \quad [1.57]$$

In the absence of an applied RF field the magnetization vector does not move at all in the rotating reference frame, as opposed to the laboratory frame that has a rotating transverse component. Second, consider the trajectory of the magnetization vector under the influence of a RF field applied on resonance ($\Delta\omega = 0$). The laboratory frame equations describe the complex trajectory shown in Fig 1.17a while the rotating frame equations describe the much simpler and easier to visualize trajectory in Fig. 1.17b.

Since the rotating frame provides a simpler method of visualizing the magnetization vector under the influence of a RF pulse, further analysis will continue in this reference frame. The total effective magnetic field in the rotating reference frame is:

$$\mathbf{B}_{\text{eff}} = B_1^c(t)\cos(\phi_{\text{rf}})\hat{\mathbf{i}}' - B_1^c(t)\sin(\phi_{\text{rf}})\hat{\mathbf{j}}' + \left(B_0 - \frac{\omega_{\text{rf}}}{\gamma}\right)\hat{\mathbf{k}}' \quad [1.58]$$

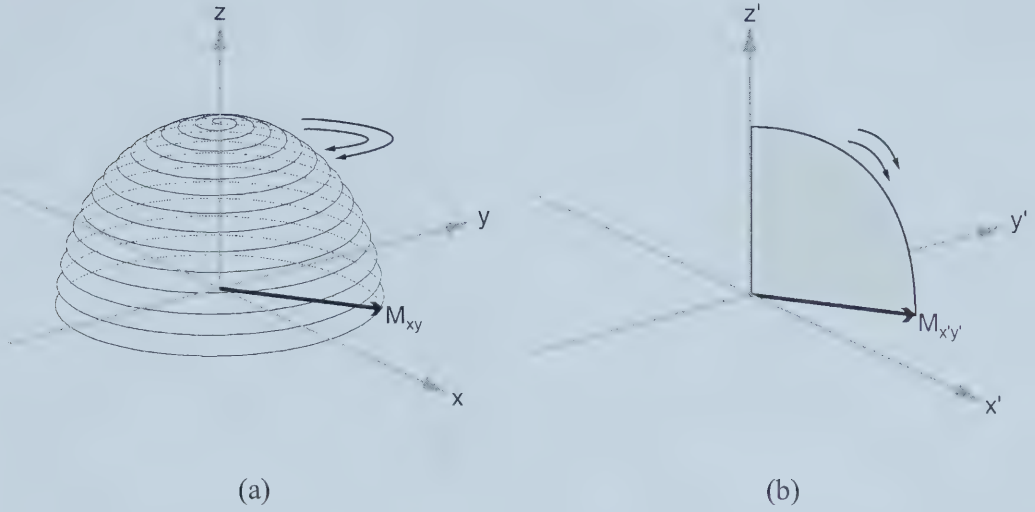


Figure 1.17: In the laboratory reference frame (a), the trajectory of the magnetization vector with an on-resonance RF field applied is a spiral trajectory. In the rotating frame (b), the trajectory is simplified to the case of the magnetization vector being flipped away from the z-axis.

While the pulse envelope is fixed the net effective magnetic field is also static. The effect of a static magnetic field on a magnetization vector has already been discussed. When the static field was $\mathbf{B}_0$, any magnetization along $\mathbf{B}_0$ was unchanged while any magnetization component that was perpendicular to $\mathbf{B}_0$ rotated about $\mathbf{B}_0$ at the Larmor frequency. In the rotating frame, any magnetization component that is parallel to $\mathbf{B}_{\text{eff}}$ will remain unchanged while any perpendicular component will rotate about $\mathbf{B}_{\text{eff}}$ at an effective frequency defined as:

$$\omega_{\text{eff}} = \gamma |\mathbf{B}_{\text{eff}}| \quad [1.59]$$

This trajectory of the magnetization vector in the rotating frame is along the surface of a cone. The cone has its tip at the origin, has the z'-axis lying along its surface and has the $\mathbf{B}_{\text{eff}}$ vector along its central axis (Fig. 1.18).

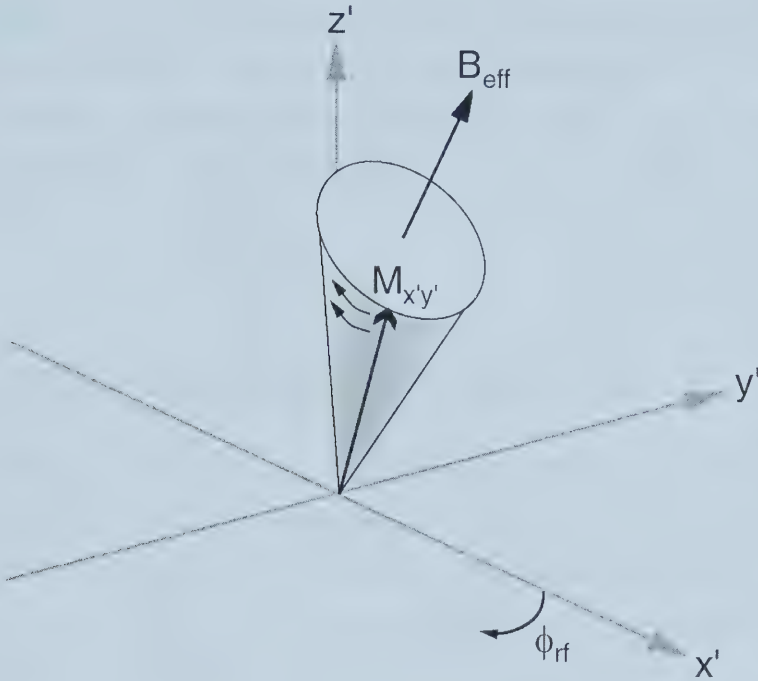


Figure 1.18: While applying a RF field, the magnetization vector rotates along the surface of a cone at an angular frequency of ω_{eff} in the direction indicated.

If the RF field continues to be applied the magnetization vector will continue all the way around the cone and return to the initial position. In order to excite the magnetization vector to a new position, a RF field is applied for a finite time to allow the magnetization to rotate partially around the cone and then the RF field is switched off.

As the RF frequency moves away from resonance, the effective magnetic field moves toward the z' -axis and the cone of precession collapses to a line along the z' -axis. In this case the magnetization vector cannot be excited into the transverse plane. As the frequency of the RF field approaches the Larmor frequency, the cone starts to expand towards the transverse plane. When the frequency of the RF field is exactly the Larmor frequency the cone of precession expands into a plane. Only when the RF field is exactly on resonance does the cone of precession contain the negative z' -axis.

A consequence of this is that the magnetization vector can only ever be inverted to reside along the negative z' -axis if the RF field is applied exactly on resonance. Since many RF pulses are applied as near resonance as possible, a more detailed examination of the magnetization vector under the influence of an on resonance RF field will be completed.

1.3.5 On Resonance Radio Frequency Pulses

When a RF pulse is applied at the Larmor frequency, the cone of precession becomes a plane and the effective magnetic field lies in the transverse plane (Fig. 1.19). Both the magnitude of the applied RF field as well as the length of time the RF field is applied define the flip angle of the RF pulse. Mathematically the flip angle θ for an on resonance RF pulse is defined as:

$$\theta = \int_{\text{pulse}} \gamma B_1^e(t) dt \quad [1.60]$$

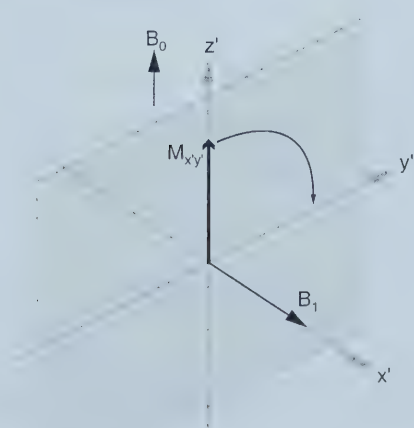


Figure 1.19: The cone of precession for an on resonance pulse along the x' -axis ($\phi_{rf} = 0^\circ$) flattens out to the $y'z'$ plane.

By adjusting the phase and the flip angle of an on resonance RF pulse the magnetization vector can be oriented in any direction. If the state of a magnetization vector is known prior to a RF pulse, then the flip angle and phase of a RF pulse are sufficient to calculate the state of the magnetization vector just after the RF pulse has been applied:

$$M_{x'}^+ = M_{x'}^- [\cos(\theta) \sin^2(\phi_{rf}) + \cos^2(\phi_{rf})] + M_{y'}^- [\sin^2(\frac{1}{2}\theta) \sin(2\phi_{rf})] - M_{z'}^- [\sin(\theta) \sin(\phi_{rf})] \quad [1.61a]$$

$$M_{y'}^+ = M_{x'}^- [\sin^2(\frac{1}{2}\theta) \sin(2\phi_{rf})] + M_{y'}^- [\cos(\theta) \cos^2(\phi_{rf}) + \sin^2(\phi_{rf})] + M_{z'}^- [\sin(\theta) \cos(\phi_{rf})] \quad [1.61b]$$

$$M_{z'}^+ = M_{x'}^- [\sin(\theta) \sin(\phi_{rf})] - M_{y'}^- [\sin(\theta) \cos(\phi_{rf})] + M_{z'}^- [\cos(\theta)] \quad [1.61c]$$

The vector $\mathbf{M}^-$ is the magnetization just prior to the RF pulse and $\mathbf{M}^+$ denotes the magnetization vector just after the RF pulse. Equation [1.61] describes the effect of an on resonance RF magnetic field on a magnetization vector.

1.4 Gradient Magnetic Fields

1.4.1 Definition of a Gradient

A gradient magnetic field, or simply a gradient, is a magnetic field that is spatially dependent. All gradient magnetic fields considered in this thesis will be linear and polarized along the direction of $\mathbf{B}_0$ as described below:

$$\mathbf{b}(\mathbf{r}) = \mathbf{G}_r \cdot \mathbf{r} \hat{\mathbf{k}} \quad [1.62]$$

The gradient strength per unit distance is $\mathbf{G}_r$ and $\mathbf{r}$ is the position where the gradient field is being observed. The net magnetic field experienced by a spin is then:

$$\mathbf{B} = [0, 0, B_0 + \mathbf{G}_r \cdot \mathbf{r}] \quad [1.63]$$

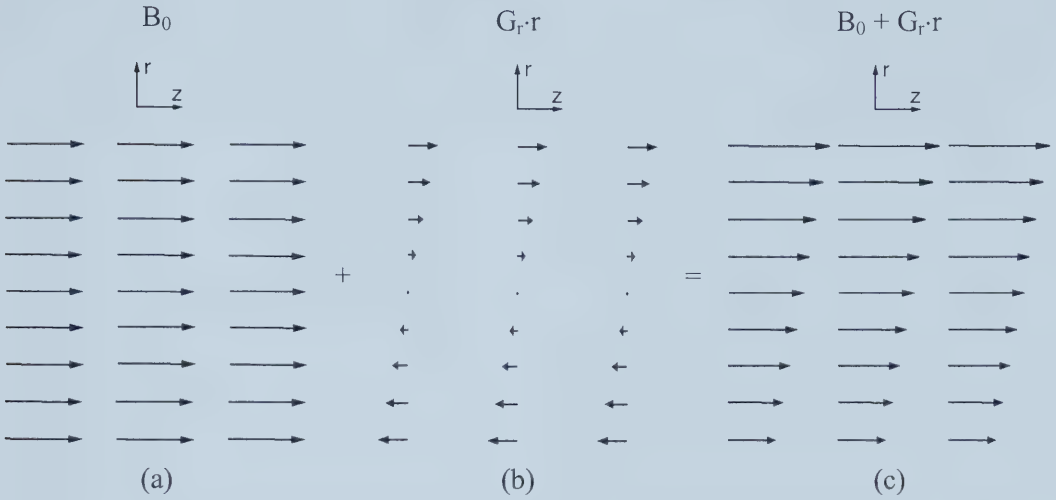


Figure 1.20: The resulting magnetic field in (c) is the sum of the (a) static magnetic field B_0 and (b) the gradient magnetic field (exaggerated for illustration purposes).

The magnitude of the gradient field is typically much less than the main static magnetic field. For example, on the 3.0 T MR scanner used to acquire data for this thesis, the three gradients each have a maximum strength of 40 mT/m. If the maximum gradient in each of the three coordinate directions were applied and the gradient field was observed 50 cm from the center of the magnet, the gradient field would still only be 2% of the main static magnetic field. As a result, even though a gradient has both positive and negative values, the net magnetic field is still always a positive value (Fig. 1.20).

The equations of motion describing a magnetization vector in a static magnetic field are still valid when gradients are applied, however, the precession frequency of the transverse magnetization is now spatially dependent (Fig. 1.21).

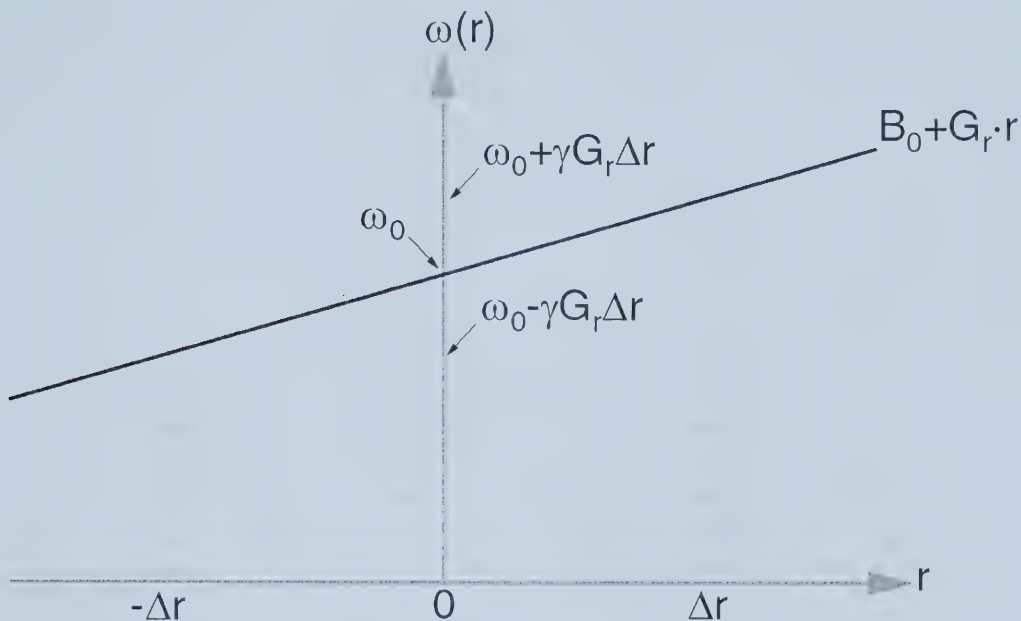


Figure 1.21: The precessional frequency of a magnetization vector with a gradient magnetic field applied depends on the spatial location of the magnetization.

Only a magnetization vector at the center of the gradient field (isocenter) will precess at ω_0 while other magnetization vectors will have angular frequencies given by their position as:

$$\omega(\mathbf{r}) = \omega_0 + \gamma \mathbf{G}_r \cdot \mathbf{r} \quad [1.64]$$

A gradient may be turned off and on and applied at varying strengths during a NMR experiment. The behavior of a gradient during an experiment is expressed with a gradient diagram. Figure 1.22 shows a simple gradient diagram and the corresponding magnetic field expected.

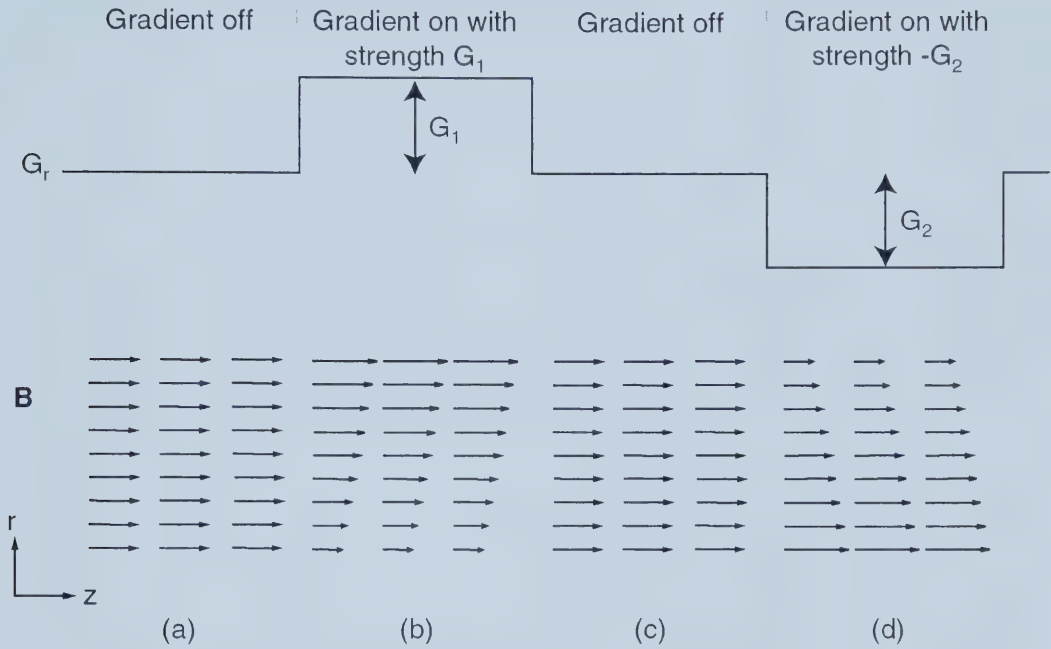


Figure 1.22: A gradient diagram containing two gradient pulses. During (a) and (c) there are no applied gradients and the magnetic field is the uniform $\mathbf{B}_0$ field. During (b) and (d), a gradient pulse is applied and the resulting magnetic field is a superposition of $\mathbf{B}_0$ and the gradient field. The strength of the gradient fields has been exaggerated for illustration purposes.

1.4.2 Gradient Moments

The position of a spin is not necessarily static during a MR experiment and is described as:

$$\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}t + \frac{1}{2}\mathbf{a}t^2 + \dots \quad [1.65]$$

The equation of motion can be generalized in the form:

$$\mathbf{r}(t) = \sum_n \frac{1}{n!} \mathbf{r}_n t^n \quad [1.66]$$

where $\mathbf{r}_0$ is the initial position of a moving spin, $\mathbf{r}_1$ the initial velocity, $\mathbf{r}_2$ the initial acceleration and so on. The n^{th} term in the series is called the n^{th} moment of motion (59). The transverse component of a magnetization vector will acquire an additional phase ϕ_G in the presence of a gradient field given as:

$$\phi_G = \gamma \int_0^{\tau_G} \mathbf{G}_r(t) \cdot \mathbf{r}(t) dt \quad [1.67]$$

The total phase acquired can be decomposed into the phase due to each moment of motion:

$$\phi_G = \sum_n \phi_{Gn} \quad [1.68]$$

where ϕ_{Gn} is the phase shift caused by the n^{th} moment of motion. The phase accumulated by the n^{th} moment of motion is expanded as:

$$\phi_{Gn} = \frac{1}{n!} \mathbf{r}_n \cdot \int_0^{\tau_G} t^n \mathbf{G}_r(t) dt \quad [1.69]$$

The integral part of the phase equation is the n^{th} moment of the gradient function $\mathbf{G}_r(t)$ (6).

1.4.3 Gradient Moment Nulling

If the gradient waveform is selected such that:

$$\int_0^{\tau_G} t^n G_r(t) dt = 0 \quad [1.70]$$

then:

$$\phi_{Gn} = 0 \quad [1.71]$$

and the n^{th} moment of the gradient is said to be nulled. The technique of selecting an appropriate waveform to satisfy Eq. [1.70] is called gradient moment nulling (60). If the n^{th} moment of the gradient is nulled, then a spin will not have any gradient induced phase shifts due to the n^{th} moment of motion. For example, a nulled 0^{th} moment implies that the initial position of a spin will not lead to any phase shifts while a nulled 1^{st} moment means there will be no gradient induced phase shifts caused by the initial velocity of the spin. In many MR applications the tissue is assumed to either be stationary or to have an approximately constant velocity. Consequently only the 0^{th} and 1^{st} gradient moments are normally given attention.

1.4.4 0^{th} Moment Nulling

The 0^{th} gradient moment arises from an initial displacement of a spin from the center of the gradient field. If a gradient waveform has no 0^{th} gradient moment then a stationary spin will have no gradient induced phase shift regardless of initial position.

One method of obtaining a nulled 0th gradient moment is to select a gradient waveform such that:

$$\int_0^{\tau_G} G_r(t) dt = 0 \quad [1.72]$$

With this method the only property required for a gradient to have no 0th gradient moment is for the total area under the gradient be zero. A nulled 0th gradient moment can also be achieved with the use of a RF refocus pulse. Using this second method, two gradient pulses with identical areas separated by a π -refocus pulse will have a nulled 0th gradient moment. Furthermore, it is also possible to combine the two nulling techniques such that some of the gradient area is nulled before a RF refocus pulse and the rest is nulled after.

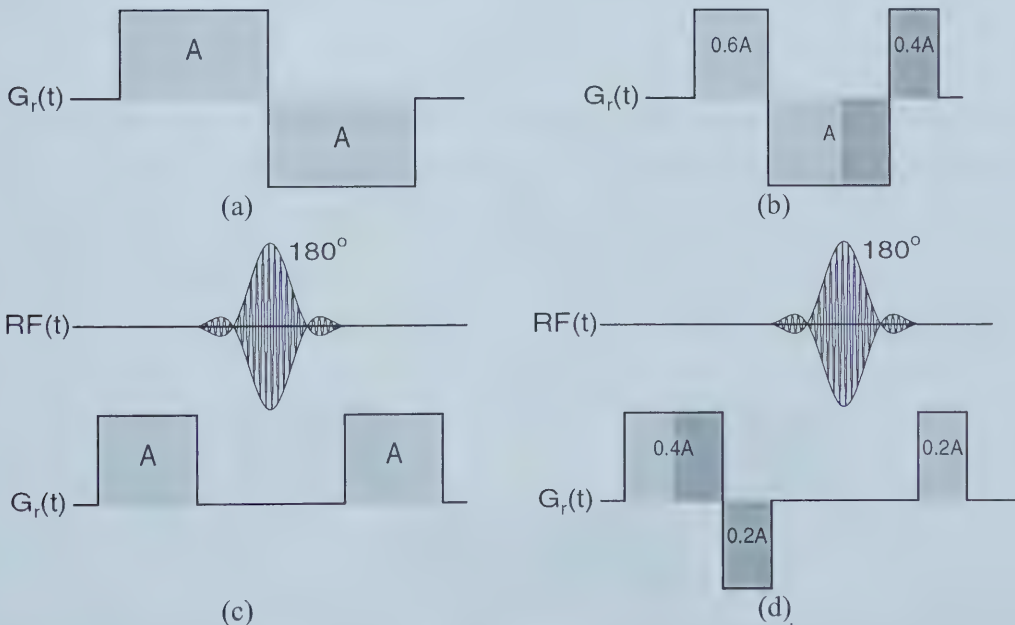


Figure 1.23: All of the gradient waveforms shown have nulled 0th gradient moments. In (a-b) the nulling is a result of the total gradient area being zero. In (c-d) a RF refocus pulse is used to invert the phase acquired from any prior gradients, effectively negating the gradient area.

1.4.5 1st Moment Nulling

The 1st gradient moment arises from an initial velocity of a spin. If a gradient waveform has no 1st gradient moment then a spin with a constant velocity will have no gradient induced phase shift due to motion. To null the 1st gradient moment, the gradient waveform should satisfy the following integral:

$$\int_0^{\tau_G} t G_r(t) dt = 0 \quad [1.73]$$

The velocity of the spin motion causes gradients that occur later in time to be more heavily weighted than those that occur earlier. Unlike the 0th moment, it is no longer sufficient to simply have an average gradient area of zero. This is an important point as bipolar gradients (two equal lobes with opposite orientations) are frequently utilized in NMR sequences.

An endless number of gradient and RF combinations can be used to create a nulled 1st gradient moment. The simplest and perhaps most common is the “1-2-1” gradient shown in Figure 1.24. In addition to having no 1st gradient moment, this waveform also has no 0th gradient moment.

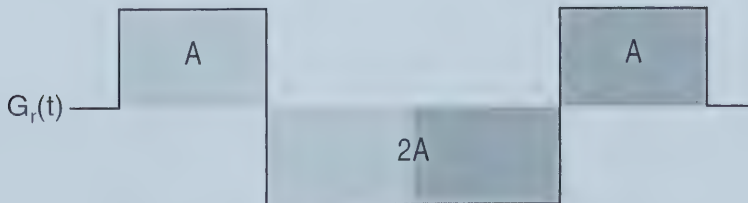


Figure 1.24: A “1-2-1” gradient has three lobes with the middle lobe being twice as large as the outside two. This configuration nulls both the 0th and 1st gradient moments.

1.5 Magnetic Resonance Imaging

This section briefly outlines some of the basic steps required to obtain a MR Image. A more in depth study of MRI can be found in the following resources (2, 5, 6, 8, 61).

1.5.1 Slice Selection

Tomography is a method of acquiring images of the internal structures of a solid object without physically cutting the object open. The term *tomos* comes from the Greek word for “cut”. The details of how a MR image may be obtained from a particular cut or slice through an object are contained in the process of slice selection. Slice selection involves the simultaneous application of a gradient magnetic field in the slice direction and a RF pulse to selectively excite a slice of spins.

Initially it will be assumed that a RF pulse may only excite a magnetization vector if it is applied on resonance. Referring to the Fourier transform pairs in Table 1.5, a RF pulse containing only a single spectral frequency requires an infinitely long sinusoidal pulse.

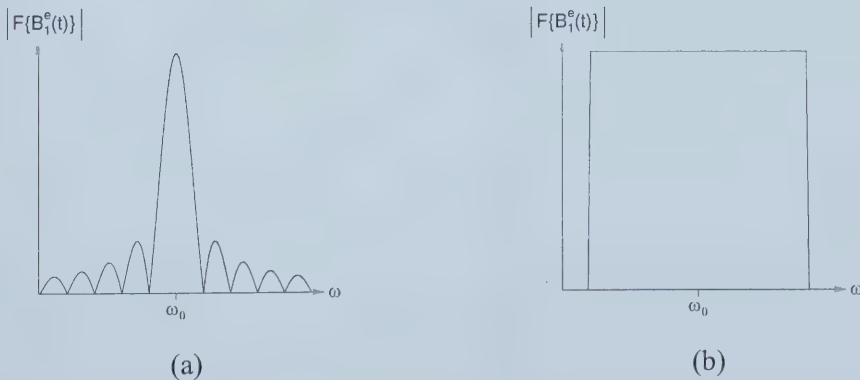


Figure 1.25: The spectral content of a RF pulse with a square envelope (a) and a sinc envelope (b) obtained using the Fourier transform.

As a result, any finite RF pulse applied in a MRI experiment will therefore have a distribution of spectral frequencies and a corresponding bandwidth (BW_{rf}). The Fourier transform may be applied to the RF pulse to extract the spectral content of the RF pulse.

The application of the slice gradient during the RF pulse will cause a linear distribution of angular precession frequencies across the slice given by Eq. [1.64]. Each spectral component of the RF pulse will be able to excite a magnetization vector that has a corresponding angular frequency. A RF pulse containing a range of frequencies will excite a range of magnetization vectors that have their precessional frequencies within the BW of the RF pulse. The thickness of the excited slice (TH) is therefore determined by both the bandwidth of the RF pulse and the strength of the slice gradient G_{ss} :

$$TH = \frac{BW_{rf}}{\gamma G_{ss}} \quad [1.74]$$

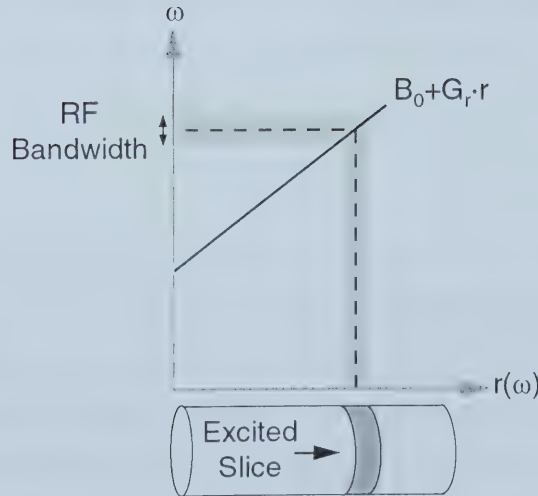


Figure 1.26: With an applied gradient field there is a linear distribution of precessional frequencies across the sample. Only the frequencies within the bandwidth of the RF pulse will be excited during the application of the RF pulse.

The flip angle received by each magnetization vector will depend on the strength of the corresponding spectral component of the RF pulse as well as the pulse length. The distribution of flip angles across a slice is called the slice profile and is then proportional to the spectrum of a RF pulse obtained with the Fourier transform. Using the Fourier transform to predict a slice profile is simple but not perfectly correct. The assumption that only an on resonance RF pulse may excite a magnetization vector is false. Spin systems behave nonlinearly during excitation and the Bloch equations presented in Eq. [1.55] must be solved to get an accurate slice profile. However, for small flip angles, the Fourier transform method is a suitable choice in estimating pulse profiles.

If the FID signal were measured after the RF and gradient pulses were turned off the resulting signal would appear smaller than expected. During the application of the RF pulse and slice gradient, magnetization vectors at different spatial locations had different angular and excitation frequencies. Accordingly, the phase of each magnetization vector at the end of the RF pulse will be spatially dependent as indicated by Eq. [1.67]. Since the FID signal is a superposition of the *emf* induced by each magnetization vector, any phase distribution will lead to destructive interference of the *emf* signals. In general the phase distribution is intricate and must be found through solutions of the Bloch equations.

Restoring the FID signal back to a maximum value requires the magnetization vectors to be brought back into phase with each other. Fortunately, in many cases the phase distribution is approximately linear and will be nearly eliminated with the application of a second gradient pulse. To approximate the linear distribution of spins presume that the slice is excited instantaneously at a time $t = 0$ as shown in Fig. 1.27.

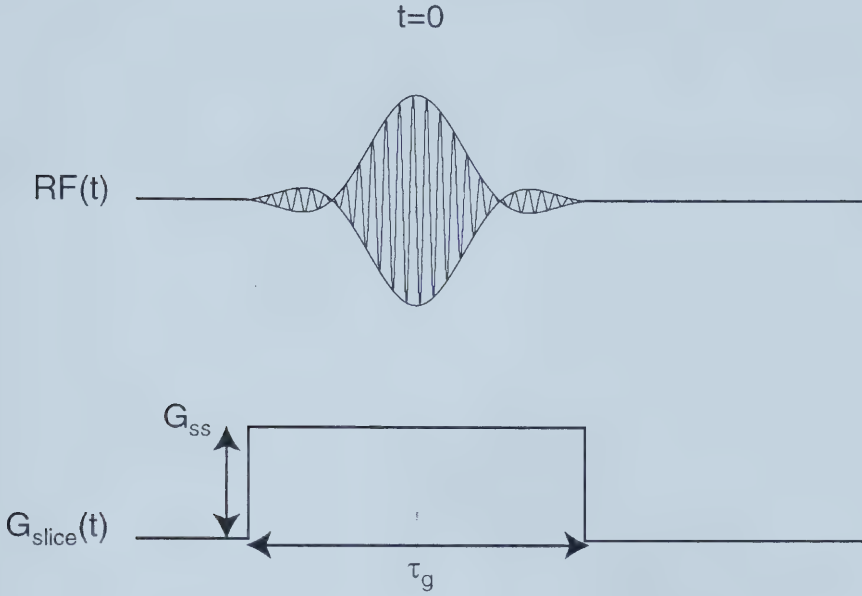


Figure 1.27: To approximate the phase distribution over the slice, the magnetization vectors are presumed to be instantaneously excited at the middle of the RF pulse ($t = 0$).

At $t = 0$ all the magnetization vectors are excited into the transverse plane and are all in phase. For the duration of the second half of the slice gradient, the additional phase acquired by each magnetization vector is:

$$\phi_g = -\gamma A_g r_s \quad [1.75]$$

where r_s is the position of the magnetization vector along the direction of the slice gradient and A_g is half the area under the slice select gradient:

$$A_g = \frac{1}{2} G_{ss} \tau_g \quad [1.76]$$

To rewind the phase of the magnetization vectors so that they regain phase coherence, a second slice gradient with an area of $-A_g$ is applied (Fig. 1.28).

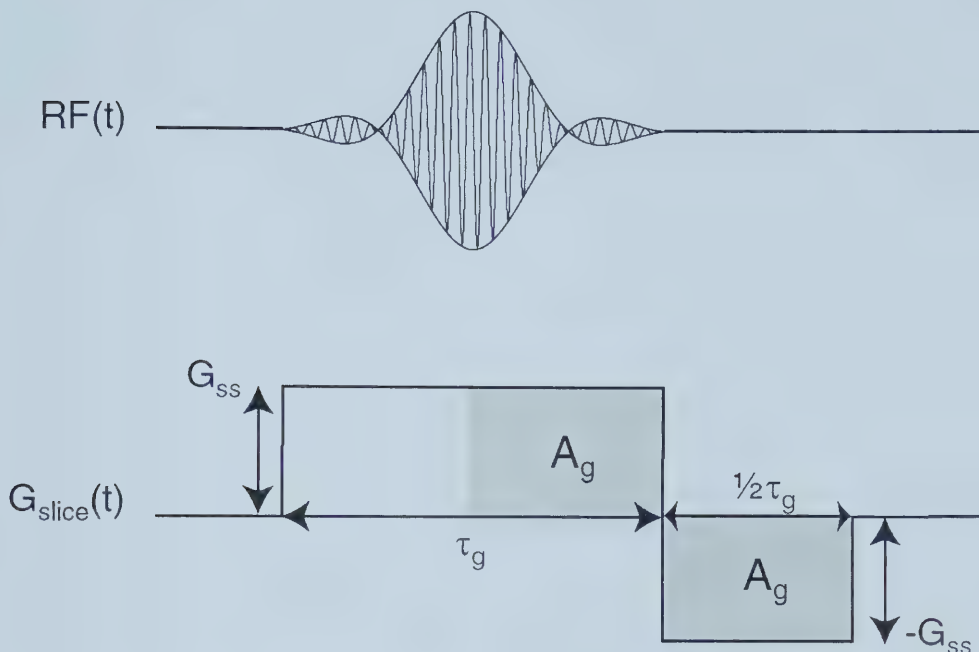


Figure 1.28: A slice of spins will be selectively excited and then rephased with the following slice gradient waveform.

Slice selection can therefore be used to localize the FID signal to within a slice or cut of the sample. However, spatial localization within the slice must still be encoded into the FID signal to obtain a MR Image.

1.5.2 Frequency Encoding

Frequency encoding is similar to slice selection except that a gradient is applied during acquisition of a FID signal rather than during the RF pulse.

During data acquisition, the application of the read gradient causes magnetization vectors at different spatial locations along the applied gradient to have different precessional frequencies. Figure 1.29 illustrates how three magnetization vectors at different spatial locations have varying angular precessional frequencies.

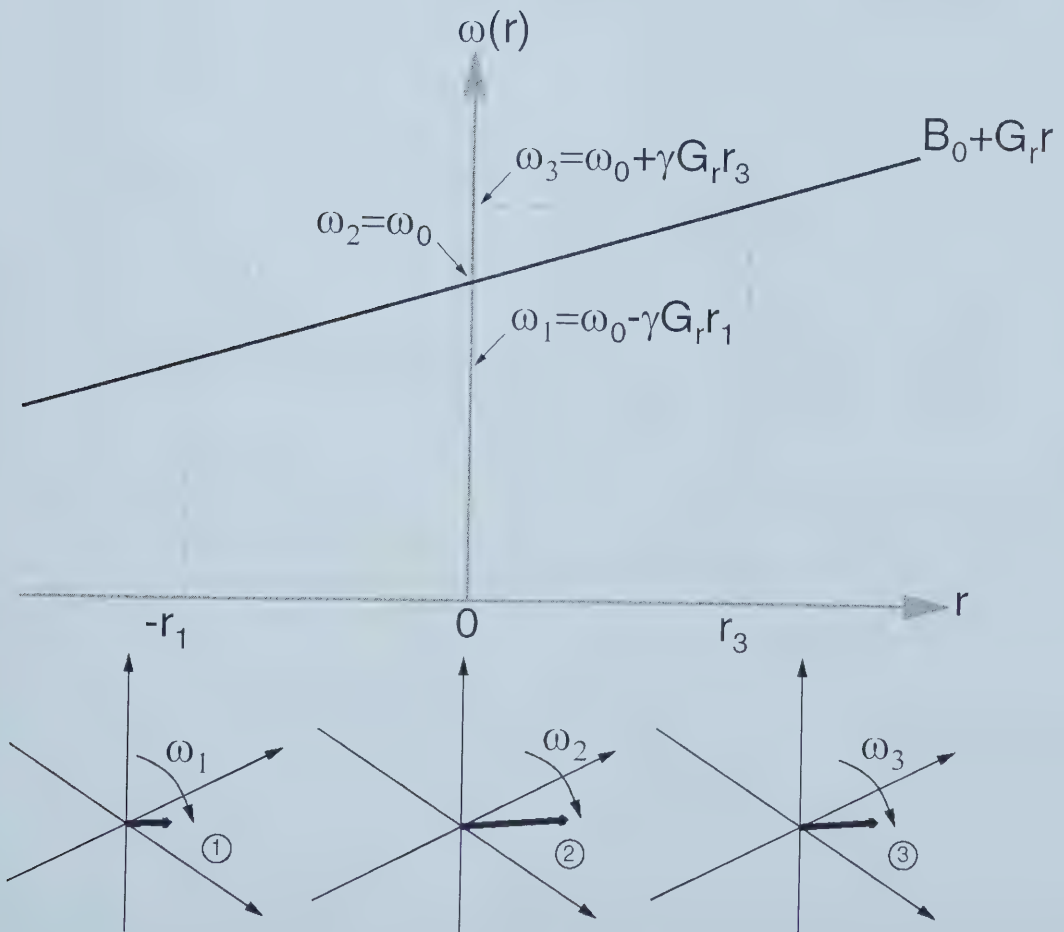


Figure 1.29: With the application of a read gradient, each of the magnetization vectors shown will have a different spatially dependent precessional frequency.

The resulting FID signal is a superposition of the induced *emf* from each of these magnetization vectors.

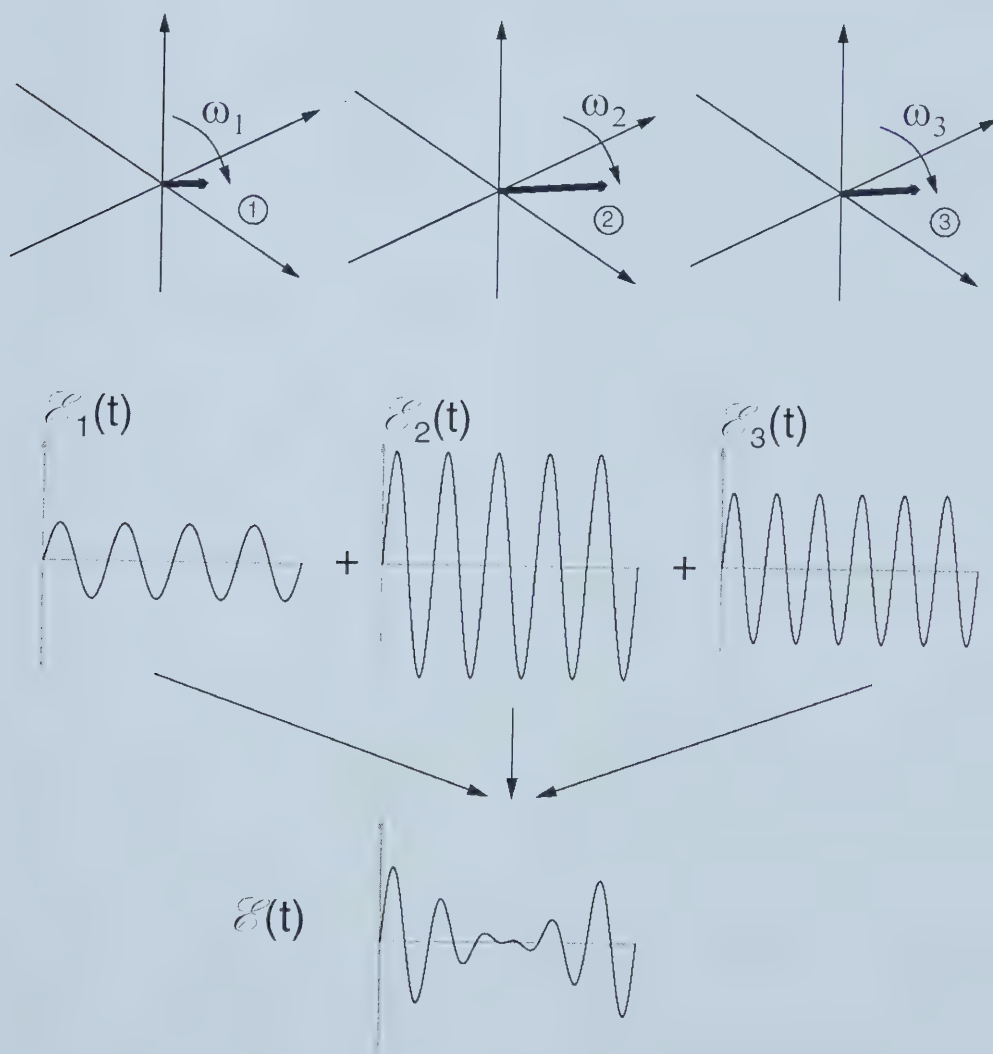


Figure 1.30: The strength of each *emf* signal is proportional to the magnitude of the corresponding magnetization vector. The FID signal observed is a superposition of each of the magnetization vectors.

The spectrum of the FID signal may be obtained with the use of the Fourier transform. The magnitude and frequency of each spectral component in the spectrum is used to calculate the location and magnitude of the contributing magnetization vector.

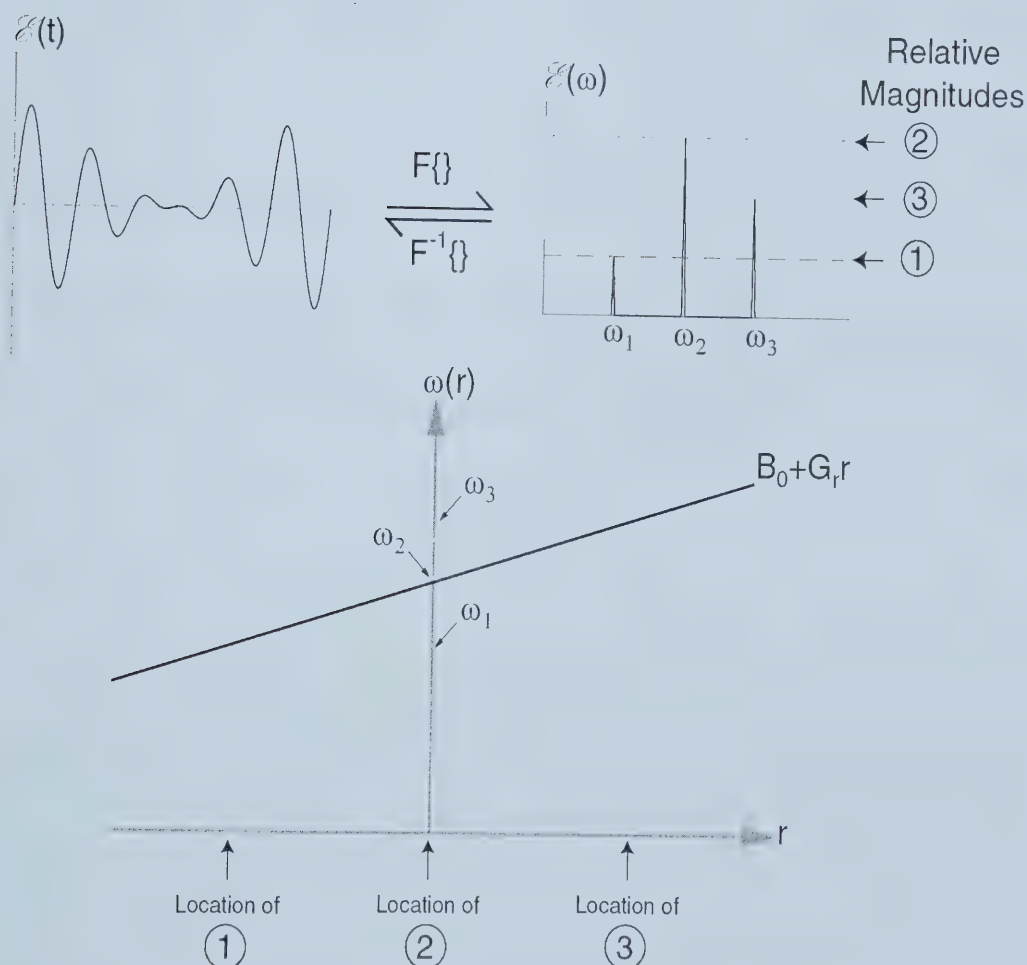


Figure 1.31: The Fourier transform is applied to the FID signal to extract the spectral information. The frequency of each spectral component determines the location of the contributing magnetization vector while the magnitude of the spectral component is proportional to the magnitude of the magnetization vector.

The total bandwidth of the acquired signal will depend on the spatial range or field of view (FOV) of the magnetization vectors and the strength of the read gradient:

$$BW_{\text{read}} = \gamma G_r \text{FOV}_r \quad [1.77]$$

To prevent aliasing of the sampled FID signal the Nyquist criteria places an upper limit on the sampling time Δt_{read} of:

$$\Delta t_{\text{read}} \leq \frac{2\pi}{\gamma G_r \text{FOV}_r} \quad [1.78]$$

Multiple gradients may not be utilized simultaneously to localize a magnetization vector in more than one dimension using frequency encoding. To provide spatial information for other dimensions, another method of spatial encoding must be utilized.

1.5.3 Phase Encoding

If a gradient is applied to a system of spins for finite time period the magnetization vectors will acquire a spatially dependent phase distribution (Fig. 1.32). Since the phase acquired depends on the position of the magnetization vector, spatial information has been encoded into the system through the phase of the magnetization vectors. This process of localization is called phase encoding and the applied gradient is called the phase encode gradient.

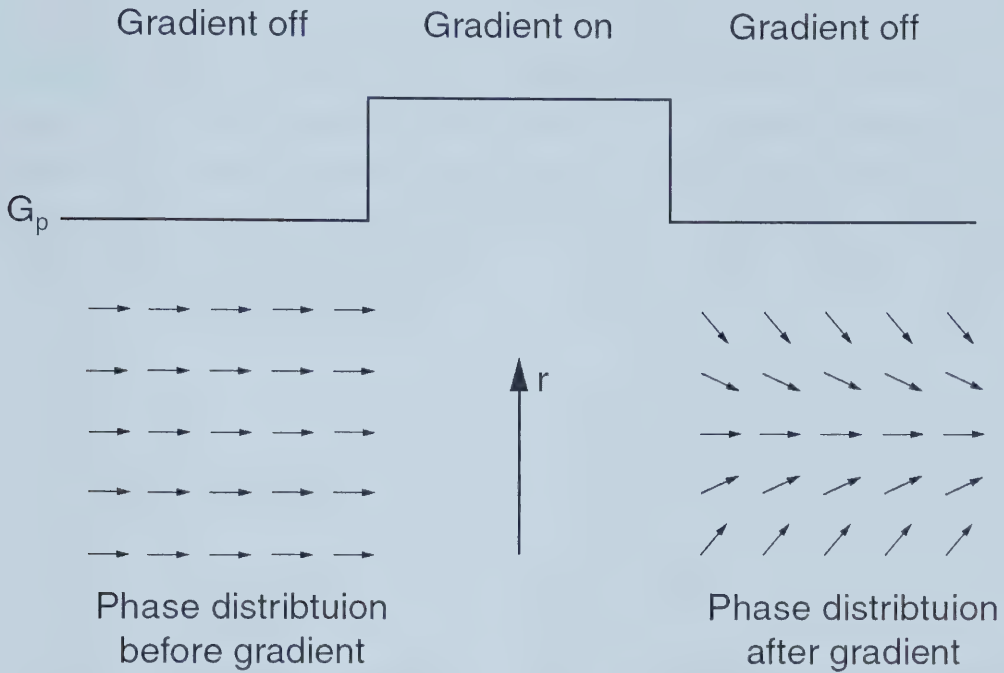


Figure 1.32: During the application of a gradient pulse, the magnetization vectors will acquire a spatially dependent phase distribution.

Since the FID signal is not continuously sampled, the phase of a magnetization vector is only known at discrete time points. The manner in which the phase was accumulated between these sampling points is not known. For example, a magnetization vector that acquires no phase at all will appear identical to a magnetization vector that made exactly one full revolution (2π) between two successive time points. The phase encode gradient strength that can be applied without causing this aliasing problem is the phase encode increment G_{pinc} and is calculated as:

$$G_{\text{pinc}} = \frac{2\pi}{\gamma \tau_p \text{FOV}_p} \quad [1.79]$$

The increment gradient strength is not the maximum phase encode gradient that is applied, but rather the largest increment in gradient strength allowed between adjacent encode steps. The methods of phase encoding and frequency encoding may be employed to completely localize a magnetization vector in a NMR experiment. To understand how these methods are used to localize the magnetization vectors from an entire slice, a mathematical description of the FID signal undergoing phase and frequency encoding is required.

1.5.4 Signal Equation

To see how phase and frequency encoding can be used to image an ensemble of spins, an equation describing the received FID signal is needed. Each magnetization vector in the sample will contribute to the induced signal. If constant gain and phase factors are ignored then the total received signal without any applied gradients is:

$$s(t) = \int_{\text{volume}} M_{xy}(\mathbf{r}, t) d\mathbf{r} \quad [1.80]$$

Expanding the equation into Cartesian components gives:

$$s(t) = \iiint_{x,y,z} M_{xy}(x, y, z, t) dx dy dz \quad [1.81]$$

With the application of gradients for either frequency or phase encoding, the total received signal in the rotating frame becomes:

$$s(t) = \iiint_{x,y,z} M_{xy}(x, y, z) \exp \left[-i\gamma \int_0^{t_0} (G_x x + G_y y + G_z z) dt \right] dx dy dz \quad [1.82]$$

Equation [1.82] resembles the Fourier transform given in Eq. [1.31]. To help with the comparison let the following definitions be made:

$$k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau) d\tau \quad [1.83a]$$

$$k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau) d\tau \quad [1.83b]$$

$$k_z(t) = \frac{\gamma}{2\pi} \int_0^t G_z(\tau) d\tau \quad [1.83c]$$

Using the definitions above, the signal equation is now expressed as:

$$s(t) = \int \int \int M_{xy}(x, y, z) \exp[-i2\pi(k_x(t)x + k_y(t)y + k_z(t)z)] dx dy dz \quad [1.84]$$

Doing a one-to-one correspondence with the Fourier transform equation, Eq. [1.84] is:

$$s(t) = \tilde{M}_{xy}[k_x(t), k_y(t), k_z(t)] \quad [1.85]$$

where,

$$\tilde{M}_{xy}(k_x, k_y, k_z) = F\{M_{xy}(x, y, z)\} \quad [1.86]$$

Thus k_x , k_y and k_z all have units of spatial frequency such as radians/meter. At any given time the signal $s(t)$ is equal to a particular value of the Fourier transform of M_{xy} . The location of this value in the Fourier transform space (called k -space) is controlled by the application of localization gradients given by Eq. [1.83].

Consequently, with the appropriate selection of localizing gradients between samples of $s(t)$, a trajectory of data points through k-space can be filled. When a sufficient amount of k-space data has been collected the inverse Fourier transform is applied to the k-space data to recover the distribution of magnetization vectors and the corresponding MR Image.

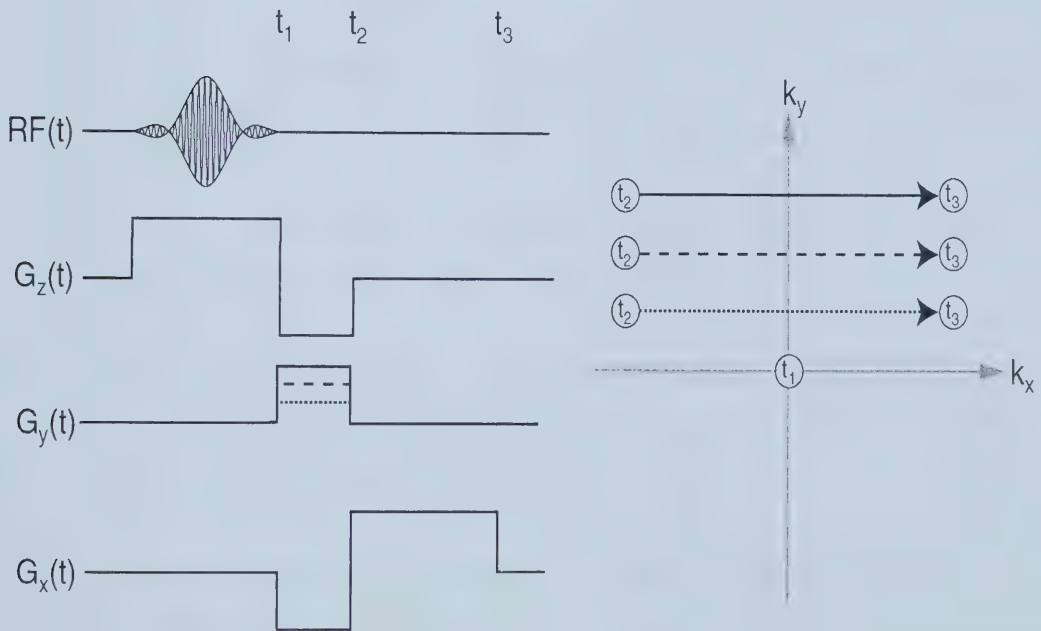


Figure 1.33: Example of a basic 2D Fourier transform imaging sequence. At time t_1 , the magnetization vectors are excited from the z-axis into the transverse plane. Between t_1 and t_2 , the negative slice gradient corrects the phase distribution the magnetization vectors acquired during excitation. The phase gradient (applied in the y-direction) has a variable strength that is used to set the k_y position in k-space. During this same interval, the negative read gradient (x-direction) positions k_x . The data is sampled between t_2 and t_3 . During data acquisition, the read gradient is turned on such that k_x moves across a line in k-space while data is being sampled. At the end of t_3 , one line of data has been acquired in k-space. To acquire other data lines, the entire experiment is started again and a different phase encode value is used to move the line of data to a new k_y position in k-space. In this manner, the segment may be repeated to fill a desired region of k-space with data.

1.6 Bibliography

1. Abragam A. The Principles of Nuclear Magnetism: The International Series of Monographs on Physics. Oxford: Clarendon Press; 1961. 599 pages.
2. Brown MA and Semelka RC. MRI Basic Principles and Applications. New York: John Wiley & Sons Inc.; 1995.
3. Ernst RR, Bodenhausen G and Wokaun A. Principles of Nuclear Magnetic Resonance in One and Two Dimensions. Oxford: Oxford University Press; 1987.
4. Fukushima E and Roeder SBW. Experimental Pulse NMR: A Nuts and Bolts Approach. Reading: Addison Wesley; 1981. 539 pages.
5. Haacke EM, Brown RW, Thompson MR and Venkatesan R. Magnetic Resonance Imaging - Physical Principles and Sequence Design. New York: John Wiley & Sons, Inc.; 1999. 914 pages.
6. Liang Z and Lauterbur PC. Principles of Magnetic Resonance Imaging. New York: IEEE Press; 2000. 416 pages.
7. Slichter CP. The Principles of Magnetic Resonance. Berlin: Springer-Verlag; 1980.
8. Vlaardingerbroek MT and den Boer JA. Magnetic Resonance Imaging 2nd Edition. Heidelberg: Springer-Verlag; 1999. 481 pages.
9. The Basics of NMR. Rochester Institute of Technology: Hornak, JP. Available from: <http://www.cis.rit.edu/htbooks/nmr/>. Accessed February 2003.

10. Gerlach W and Stern O. Der experimentelle Nachweis der Richtungsquantelung. *Z Phys* 1922a; 9: 349-352.
11. Uhlenbeck GE and Goudsmit S. Ersetzung der Hypothese von unmechanischen Zwang durch eine Forderung bezüglich des inneren Verhaltens jedes einzelnen Elektrons. *Nature* 1925;
12. Bransden BH and Joachain CJ. *Introduction to Quantum Mechanics*. Harlow: Addison Wesley Longman Limited; 1989. 707 pages.
13. Estermann I and Stern O. Untersuchungen zur Molekularstrahlmethode. *Z Phys* 1933; 85: 17.
14. Griffiths DJ. *Introduction to Elementary Particles*. New York: John Wiley & Sons Inc.; 1987. 392 pages.
15. Allen PS. Some Fundamental Principles of Nuclear Magnetic Resonance. In: M. Bronskill and P. Sprawls. *The Physics of MRI - 1992 AAPM Summer School Proceedings*. Woodbury: American Institute of Physics, Inc.; 1993. 15-31
16. Mandl F. *Statistical Physics*. Trowbridge: John Wiley & Sons, Ltd.; 1971. 385 pages.
17. Feynman RP, Vernon FL Jr. and Hellwarth RW. Geometrical Representations of the Schroedinger Equation for Solving Maser Problems. *J Appl Phys* 1957; 28: 49.
18. Goldman M. *Quantum Description of High-Resolution NMR in Liquids*. Oxford: Oxford University Press; 1988.

19. Sorensen OW, Eich GW, Levitt MH, Bodenhausen G and Ernst RR. Product Operator Formalism for the Description of NMR Pulse Experiments. *Prog NMR Spec* 1983; 16: 163.
20. Wokaun A and Ernst RR. Selective Excitation and Detection in Multilevel Spin Systems: Application of Single Transition Operators. *J Chem Phys* 1977; 67: 1752.
21. Griffiths DJ. *Introduction to Electrodynamics*. Upper Saddle River: Prentice-Hall Inc.; 1989. 532 pages.
22. Cheng DK. *Field and Wave Electromagnetics*. Reading: Addison-Wesley Publishing Company; 1992. 703 pages.
23. Solomon I. Relaxation Processes in a System of Two Spins. *Phys Rev* 1955; 99: 559.
24. Bloembergen EM, Purcell EM and Pound RV. Relaxation Effects in NMR Absorption. *Phys Rev* 1948; 73: 679.
25. Bloch F. Nuclear Induction. *Phys Rev* 1946; 70: 460-474.
26. Purcell EM, Torrey HC and Pound RB. Resonance Absorption by Nuclear Magnetic Moments in a Solid. *Phys Rev* 1946; 69: 37-38.
27. Korb JP and Bryant RG. Magnetic Field Dependence of Proton Spin-Lattice Relaxation Times. *Magn Reson Med* 2002; 48: 21-26.

28. Bazelaire C, Duhamel G and Alsop D. NMR Relaxation Times in Human Abdominal Tissues at 3.0 Tesla. Proceedings of the 10th Annual Meeting of ISMRM 2002; 2292.
29. Chen L, Bernstein M, Huston J and Fain S. Measurements of T1 Relaxation Times at 3.0 T: Implications for Clinical MRA. Proceedings of the 9th Annual Meeting of ISMRM 2001; 1391.
30. Jackson JD. Classical Electrodynamics: 3rd Edition. New York: John Wiley & Sons Inc.; 1998. 808 pages.
31. Oppenheim AV and Schafer RW. Discrete Time Signal Processing. Englewood Cliffs: Prentice Hall Inc.; 1989.
32. Roden MS. Analog and Digital Communication Systems. Upper Saddle River: Prentice Hall Inc.; 1996. 560 pages.
33. Soliman SS and Mandyam DS. Continuous and Discrete Signals and Systems. Englewood Cliffs: Prentice Hall Inc.; 1990. 503 pages.
34. Brigham EO. The Fast Fourier Transform. Englewood Cliffs: Prentice-Hall; 1974.
35. Rabi II, Ramsey NF and Schwinger J. Use of Rotating Coordinates in Magnetic Resonance Problems. Rev Mod Phys 1954; 26: 167.
36. Dielectric Properties of Body Tissues at RF and Microwave Frequencies. Washington: Federal Communications Commission. Available from: <http://www.fcc.gov/fcc-bin/dielec.sh>. Accessed February 2003.

37. Krauss JD. *Electromagnetics*. New York: McGraw-Hill; 1984. p216.
38. Hopkins JA and Wehrli FW. Magnetic Susceptibility Measurement of Insoluble Solids by NMR: Magnetic Susceptibility of Bone. *Magn Reson Med* 1997; 37: 494-500.
39. Weisskoff RM and Kiihne S. MRI Susceptometry: Image Based Measurement of Absolute Susceptibility of MR Contrast Agents, and Human Blood. *Magn Reson Med* 1992; 24: 375-383.
40. Collins CM, Yang B, Yang QX and Smith MB. Numerical Calculations of the Static Magnetic Field in Three-Dimensional Multi-Tissue Models of Human Head. *Magn Reson Imaging* 2002; 20: 413-424.
41. Jin J. *Electromagnetic Analysis and Design*. New York: CRC Press; 1999. 282 pages.
42. Hoult DI. The NMR Receiver: A Description and Analysis of Design. *J Magn Reson Imaging* 2000; 12: 46-67.
43. Alsop DC, Connick TJ and Mizsei G. A Spiral Volume Coil for Improved RF Field Homogeneity at High Static Magnetic Field Strengths. *Magn Reson Med* 1998; 40: 49-54.
44. Born M and Wolf E. *Principles of Optics*, 7th Ed. Cambridge: Cambridge University Press; 1999. 986 pages.
45. Welch WJ. Reciprocity Theorems for Electromagnetic Fields Whose Time Dependence is Arbitrary. *IEEE Trans Antennas Propag* 1960; 8: 68-73.

46. Schenck JF. Radiofrequency Coils: Types and Characteristics. In: M. Bronskill and P. Sprawls. *The Physics of MRI - 1992 AAPM Summer School Proceedings*. Woodbury: American Institute of Physics, Inc.; 1993. 98-134
47. Hoult DI and Phil D. Sensitivity and Power Deposition in a High-Field Imaging Experiment. *J Magn Reson Imaging* 2000; 45: 46-67.
48. Sank VJ, Chen CN and Hoult DI. A Quadrature Coil for the Adult Human Head. *J Magn Reson* 1986; 69: 236-242.
49. Kramer J. Production of Homogeneous Fields. *Brit J Appl Phys* 1967; 18: 1815-1818.
50. Kelter JR, Carlson JW, Roos MS, Wong STS, Wong TL and Budinger TF. Electromagnetic Fields of Surface Coil In Vivo NMR at High Frequencies. *Magn Reson Med* 1991; 22: 467-480.
51. Edelstein WA, Foster TH and Schenck JF. The Relative Sensitivity of Surface Coils to Deep Lying Tissue. *SMRM Proceedings* 1985; 964-965.
52. Schenck JF, Hart HR, Foster TH, Edelstein WA and Hussain MA. High Resolution Magnetic Resonance Imaging Using Surface Coils. In: H. Kressel. *Magnetic Resonance Annual*. New York: Raven Press; 1986. 123-160
53. Schneider HJ and Dullenkopf P. Slotted Tube Resonator: A New NMR Head Probe at High Observing Frequencies. *Rev Sci Instrum* 1977; 48: 68-73.

54. Sotgiu A, Gualtieri G and Passariello R. Highly Homogeneous Circularly Polarized RF Field for Whole Body NMR Imaging. *Magn Reson Imaging* 1988; 6: 249-254.
55. Hayes CE, Edelstein WA, Schenck JF, Mueller OM and Eash M. An Efficient, Highly Homogeneous Radiofrequency Coil for Whole Body NMR Imaging. *J Magn Reson* 1985; 63: 622-628.
56. Tropp J. The Theory of the Bird Cage Resonator. *J Magn Reson* 1989; 82: 51-62.
57. Baertlein BA, Ozbay O, Ibrahim T, Lee R, Yu Y, Kangarlu A and Robitaille PML. Theoretical Model for an MRI Radio Frequency Resonator. *IEEE Trans BioMed Eng* 2000; 47: 535-546.
58. Ginsberg DM and Melchner MJ. Optimum Geometry of Saddle Shaped Coils for Generating a Uniform Magnetic Field. *Rev Sci Instrum* 1970; 41: 122.
59. Halliday D, Resnick R and Walker J. *Fundamentals of Physics*: 4th Edition. New York: John Wiley & Sons Inc.; 1993. 1306 pages.
60. Hinks RS and Constable RT. Gradient Moment Nulling in Fast Spin Echo. *Magn Reson Med* 1994; 32: 698-706.
61. Mansfield P and Morris PG. *NMR Imaging in Biomedicine*. New York: Academic Press; 1982.

2

The Spin Echo

Spin echoes, first described by Hahn in 1950, have the ability to refocus phase shifts caused by field inhomogeneities (1). Spin echo sequences are capable of providing proton density (PD), T_1 (longitudinal relaxation) weighted and T_2 (transverse relaxation) weighted images (2, 3). However, the patient discomfort associated with the long scan times of spin echo sequences can lead to slice positioning problems and motion artifacts in clinical MRI (4). This has led to the development of Rapid Acquisition with Relaxation Enhancement (RARE) or fast spin echo (FSE) sequences. These sequences acquire multiple data sets in a single segment to significantly reduce imaging times while maintaining many of the benefits of spin echo sequences (5-9). This chapter discusses the mechanisms behind spin echoes and explains how echoes are utilized in FSE sequences.

2.1 Spin Echo Mechanism

2.1.1 Definition of Spin Echoes

It has been shown in the introductory chapter that the FID signal observed in a NMR experiment has contributions from numerous spins within the field of view of the receiver coil. A maximum signal will be observed only if all the contributing magnetization vectors are in phase with each other. Any phase distribution across the sample will cause destructive interference to the FID signal and the magnetization vectors must regain phase coherence to restore the FID to a maximum (10). Except for some irreversible relaxation, the rephased FID signal will resemble the original FID and is said to be an echo of the original signal.

If the phase distribution is caused by the application of gradient magnetic fields, then the phase distribution may be undone with a negative set of gradients and the resulting echo is labeled a gradient echo. A spin echo is formed when a phase distribution caused by magnetic field inhomogeneities and chemical shift is rephased (11). Since the field inhomogeneities and chemical shift mechanisms cannot be reversed like a gradient field, another method of rephasing the magnetization vectors is required.

When the dephasing mechanism is only spatially dependent and relaxation is neglected, the phase of each magnetization vector in the rotating frame can be expressed as:

$$\phi(\mathbf{r}, t) = \phi_0 + \Delta\omega(\mathbf{r})t \quad [2.1]$$

where ϕ_0 is the initial phase common to every magnetization vector and $\Delta\omega(\mathbf{r})$ describes the spatially dependent accumulation of phase for a particular magnetization vector.

A magnetization vector that is allowed to acquire phase for a time period τ has a final phase in the rotating frame of:

$$\phi(\mathbf{r}, \tau) = \phi_0 + \Delta\omega(\mathbf{r})\tau \quad [2.2]$$

Figure 2.1 illustrates how the acquisition of a phase distribution degrades the FID signal. If the distribution of phase is spatially uniform across the sample then the FID signal will have a Sinc function modulation.

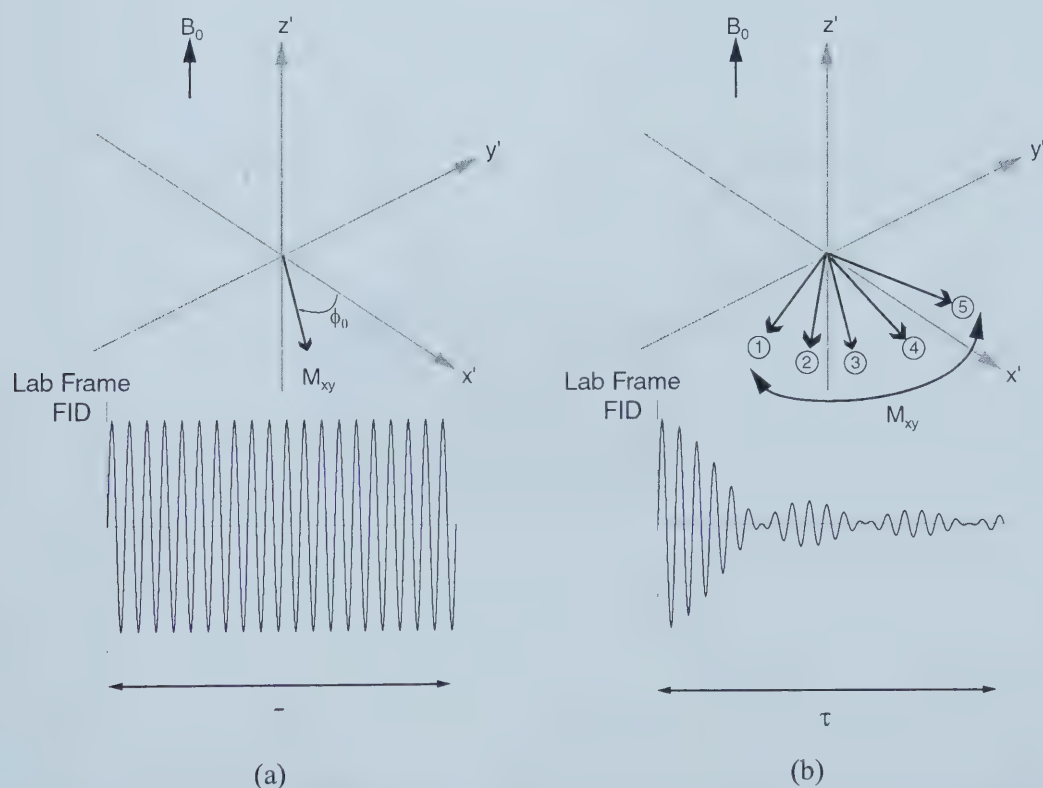


Figure 2.1: Neglecting T_2 relaxation, when all the magnetization vectors are in phase (a) the FID signal does not suffer from any degradation over time. In contrast, when the magnetization vectors acquire a uniform phase distribution over time (b) the vectors destructively interfere producing the degraded FID signal shown.

Suppose that after a time τ some event (Fig. 2.2) causes any phase offset from the initial value of ϕ_0 to become inverted. The particulars of the event itself are postponed until the next section and only the existence of the event is necessary to continue with the present analysis. The phase of a magnetization vector just after this event is then:

$$\phi(\mathbf{r}, \tau^+) = \phi_0 - \Delta\omega(\mathbf{r})\tau \quad [2.3]$$

The time independent mechanism that caused the initial phase distribution will continue to cause a magnetization vector to acquire phase according to Eq. [2.1].

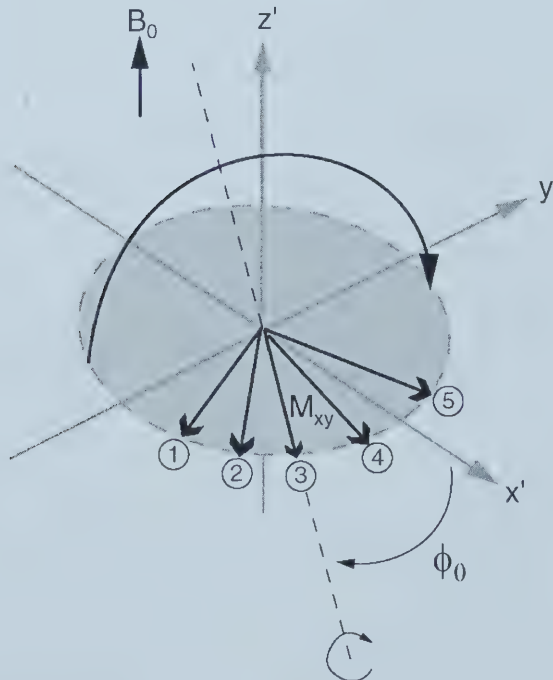


Figure 2.2: The magnetization vectors ① through ⑤ have a phase distribution acquired over the first time period τ . When the transverse plane is flipped about an axis (offset ϕ_0 from the x' -axis) the magnetization vectors have any acquired phase inverted.

The phase of a magnetization vector in the rotating frame after the event ($t = 0$) is:

$$\phi(\mathbf{r}, t) = \phi_0 - \Delta\omega(\mathbf{r})(\tau - t) \quad [2.4]$$

If the system is allowed to evolve for another time period of τ , Eq. [2.4] indicates that the phase of a magnetization vector will return to the initial value of ϕ_0 regardless of spatial location.

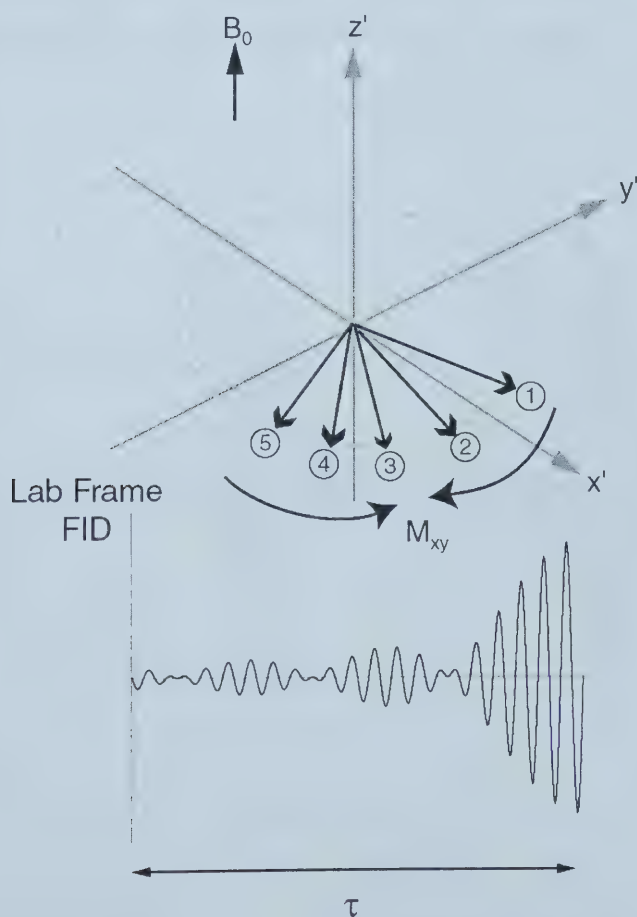


Figure 2.3: As the magnetization vectors continue to acquire phase they will rephase a time period τ after being flipped.

Once the magnetization vectors rephase after the second time period of τ , they will then continue acquiring phase and once again the FID signal will degrade. The FID signal can be rephased back to a maximum value by repeating the whole procedure. Using this method, the FID signal can be rephased to its peak value every period of 2τ (12). This period of time is denoted as the echo time and is abbreviated as TE.

The reformation of the FID signal back to its peak value is called an echo. Figure 2.4 illustrates the original FID decay and the subsequent formation of two echoes. Because T_2 relaxation has been ignored, each echo will have the same amplitude as the original FID and it appears that an infinite number of echoes can be created.

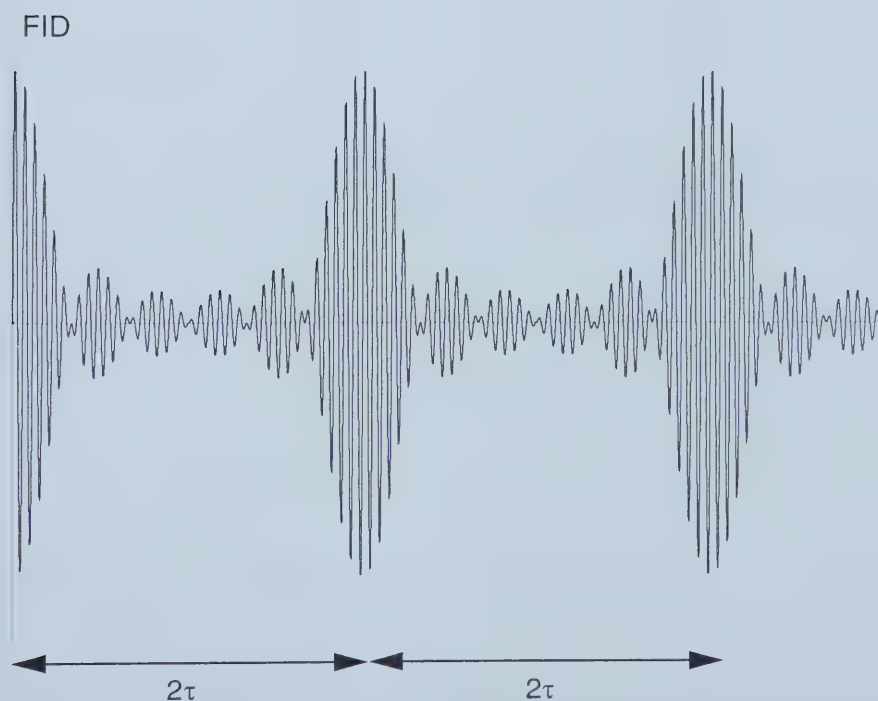


Figure 2.4: The FID signal can be repeatedly rephased every period of TE (2τ).

Transverse relaxation is not negligible in a NMR experiment and the effect of the transverse signal decay must be included. In addition, the FID in Fig. 2.4 is calculated assuming that the magnetization vectors will acquire a uniform phase distribution rather than a distribution leading to a T_2^* exponential signal decay (13, 14). Consequently, although the reversible component of relaxation is undone when an echo is formed, each successive echo will still suffer from the effects of the non-reversible T_2 relaxation.

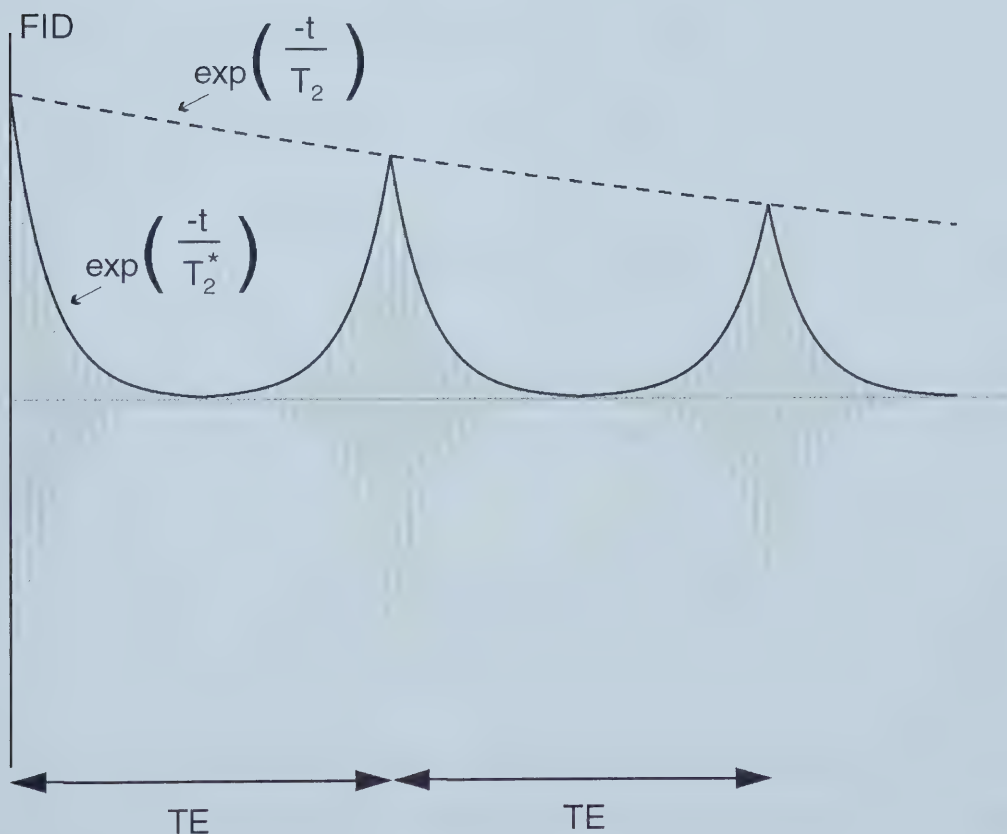


Figure 2.5: The FID signal has a decay envelope that is governed by the time constant T_2^* . This envelope has reversible (T_2') and non-reversible (T_2) components. The reversible component can be rephased and the lost signal can be brought back through an echo. Because the non-reversible component cannot be removed, the echo will not be able to recover any signal loss due to the non-reversible component.

The ability of a spin echo to refocus the phase distribution caused by local field inhomogeneities make it a desirable mechanism in NMR experiments. The theory presented in this section relied on the existence of a particular refocus event that was able to manipulate the phase of the magnetization vectors. The next section will demonstrate how a RF pulse is used to create a spin echo.

2.1.2 Generating Spin Echoes with RF Pulses

Consider a magnetization vector in the rotating frame at a time $t = 0$. The initial magnetization is:

$$M_{x'y'}(0) = M_{x'}(0) + iM_{y'}(0) \quad [2.5]$$

In a time period of τ the magnetization vector will acquire an additional phase $\Delta\phi$. Neglecting the effects of T_2 , the transverse magnetization components at the end of that time period are:

$$M_{x'}(\tau) = M_{x'}(0)\cos(\Delta\phi) + M_{y'}(0)\sin(\Delta\phi) \quad [2.6a]$$

$$M_{y'}(\tau) = -M_{x'}(0)\sin(\Delta\phi) + M_{y'}(0)\cos(\Delta\phi) \quad [2.6b]$$

Next a 180° RF pulse with an arbitrary phase ϕ_{rf} is applied on resonance to the system. Utilizing Eq. [1.44], the transverse magnetization components just after the RF pulse ($t = \tau^+$) are:

$$M_{x'}(\tau^+) = M_{x'}(\tau)\cos(2\phi_{rf}) + M_{y'}(\tau)\sin(2\phi_{rf}) \quad [2.7a]$$

$$M_{y'}(\tau^+) = M_{x'}(\tau)\sin(2\phi_{rf}) - M_{y'}(\tau)\cos(2\phi_{rf}) \quad [2.7b]$$

Combining the results of Eq. [2.6] and [2.7], the transverse magnetization components just after the RF pulse are expanded as:

$$M_{x'}(\tau^+) = M_{x'}(0)\cos(\Delta\phi + 2\phi_{rf}) + M_{y'}(0)\sin(\Delta\phi + 2\phi_{rf}) \quad [2.8a]$$

$$M_{y'}(\tau^+) = M_{x'}(0)\sin(\Delta\phi + 2\phi_{rf}) - M_{y'}(0)\cos(\Delta\phi + 2\phi_{rf}) \quad [2.8b]$$

Following the RF pulse the magnetization vector will again acquire a phase of $\Delta\phi$ during the second time period of τ . At the end of the second time period ($t = TE$) the transverse magnetization is:

$$M_{x'}(TE) = M_{x'}(0)\cos(2\phi_{rf}) + M_{y'}(0)\sin(2\phi_{rf}) \quad [2.9a]$$

$$M_{y'}(TE) = M_{x'}(0)\sin(2\phi_{rf}) - M_{y'}(0)\cos(2\phi_{rf}) \quad [2.9b]$$

Equation [2.9] indicates that the final transverse magnetization is independent of $\Delta\phi$. If an ensemble of magnetization vectors undergoes this sequence of events and each magnetization vector has a different value of $\Delta\phi$, all the magnetization vectors will regain phase coherence at the echo time (TE). This demonstrates that a 180° RF pulse can be used to rephase a distribution of magnetization vectors caused by static field inhomogeneities or chemical shift. Because this type of RF pulse is able to refocus the magnetization vectors and produce a spin echo, it is referred to as a refocusing pulse.

A sequence that utilizes a full excitation pulse ($\pi/2$ or 90° flip angle) and is followed by a train of π refocus pulses (180° flip angle) with the same phase as the excitation pulse is labeled a Carr-Purcell (CP) sequence (15). In a NMR experiment the amplitude of the refocus pulses will always vary to some degree within the sample. Repeated application of non-perfect refocus pulses will lead to cumulative errors that can be lessened with the use of a phase cycling scheme. This correction, proposed by Meiboom and Gill, is

implemented by shifting the phase of the RF refocus pulses by $\pi/2$ from the excitation pulse (16). For example, a Carr-Purcell-Meiboom-Gill (CPMG) with an excitation pulse along the x-axis ($\pi/2_x$) would have each refocus pulse applied along the y-axis (π_y). Despite the improved performance of a CPMG sequence over a CP sequence, the modified sequence is not immune to the effects of varying refocus flip angles (17, 18).

It should be noted that although the magnetization vectors are rephased following a π refocus pulse, they are not necessarily located in the same position. The final phase of the refocused magnetization vectors depends on both the initial phase of the magnetization vectors and the phase ϕ_{rf} of the refocus RF pulse. This is an important consideration as the phase of the magnetization vectors plays an important role in many NMR procedures such as phase encoding. In addition, the phase of a magnetization vector is a key factor in the design of the flip angle and phase of successive RF pulses in FSE sequences (19).

2.1.3 Effect of Reduced Flip Angles on Spin Echoes

It has been shown that any phase distributions caused by static field inhomogeneities are exactly refocused with a π (180°) RF refocus pulse while fluctuating inhomogeneities result in the gradual transverse decay of the echo amplitudes. However, it is possible to partially refocus a phase distribution with a RF refocus pulse that has a flip angle reduced from π . The spin echoes resulting from reduced refocus flip angles will consequently have degraded amplitudes. To illustrate this point, consider a modified CPMG spin echo sequence containing two RF pulses. The sequence will fully excite the magnetization into the transverse plane with a $\pi/2$ excitation pulse but will instead be followed by a generalized refocus pulse flip angle of θ .

The $\pi/2$ - θ sequence has a spin echo amplitude of

$$M_{xy} = M_0 \sin^2\left(\frac{1}{2}\theta\right) e^{\frac{-TE}{T_2}} \quad [2.10]$$

where T_2 is the spin-spin relaxation constant and TE is the echo time of the spin echo (4). Equation [2.10] shows that the amplitude of the echo has a $\sin^2(1/2\theta)$ dependence on the refocus flip angle and even though the echo amplitude will be reduced, a spin echo can still be formed with a reduced flip angle refocus pulse. Chapter three of this thesis will discuss the benefits and downfalls of utilizing reduced flip angles whether by design or limitation. Benefits include the possibility of shorter RF pulses or reduced RF power deposition (20, 21). The downfalls include the possibility of reduced signal to noise (SNR), the increasingly more complex trajectory of the magnetization vectors and the possibility of generating stimulated echoes (17, 18, 22, 23).

2.1.4 Definition of Stimulated Echoes

The previous section discussed the formation of a single spin echo following a single RF refocus pulse with an arbitrary flip angle. Although each additional RF refocus pulse is able to create an additional spin echo, phase memory will lead to the creation of stimulated echoes (1, 4). Phase memory is a mechanism in which a magnetization vector is able to remember the phase accumulated while it existed in the transverse plane at some previous time period. Figure 2.6 demonstrates the creation of a stimulated echo through the mechanism of phase memory.

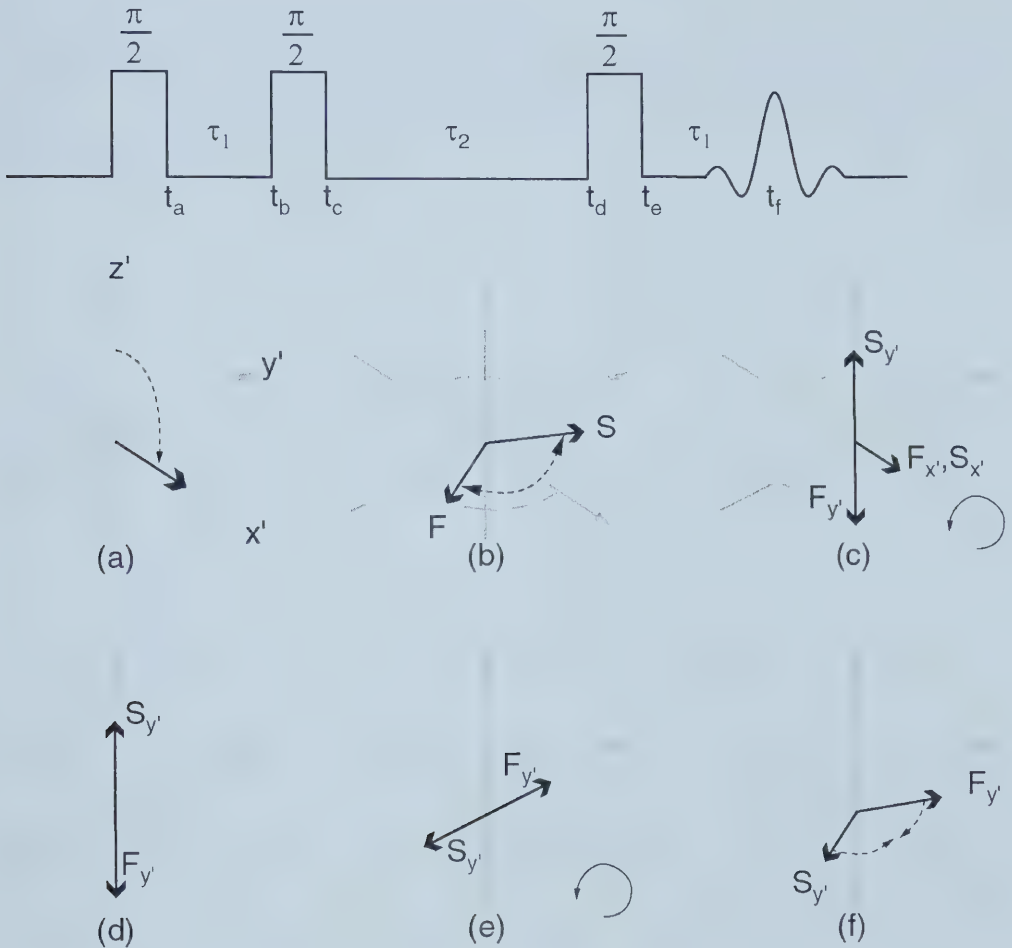


Figure 2.6: A NMR experiment with three $\pi/2$ RF pulses will create both stimulated and spin echoes. The first RF pulse excites the magnetization along the x' -axis (a). During the first time period τ_1 (b) a fast spin (F) and slow spin (S) will acquire and lose phase in the rotating frame respectively. A second RF pulse in (c) will flip the y' -components of the spins along the z' -axis leaving the x' -components in the transverse plane. For the period of time τ_2 both the transverse and longitudinal components will relax with their corresponding relaxation times. Considering just the y' -components (d), when the third RF pulse is applied, the y' -components will be brought back into the transverse plane (e). The fast spin will continue to acquire phase and the slow spin will continue to lose phase and the spins will rephase a time τ_1 later producing a stimulated echo (f). The x' -components of the magnetization vectors could possibly create a stimulated echo with the application of an additional RF refocus pulse.

A NMR experiment with three RF pulses will generate a total of five echoes (24). Figure 2.7 illustrates the five generated echoes with the specific locations and amplitudes given in Table 2.1 (25).

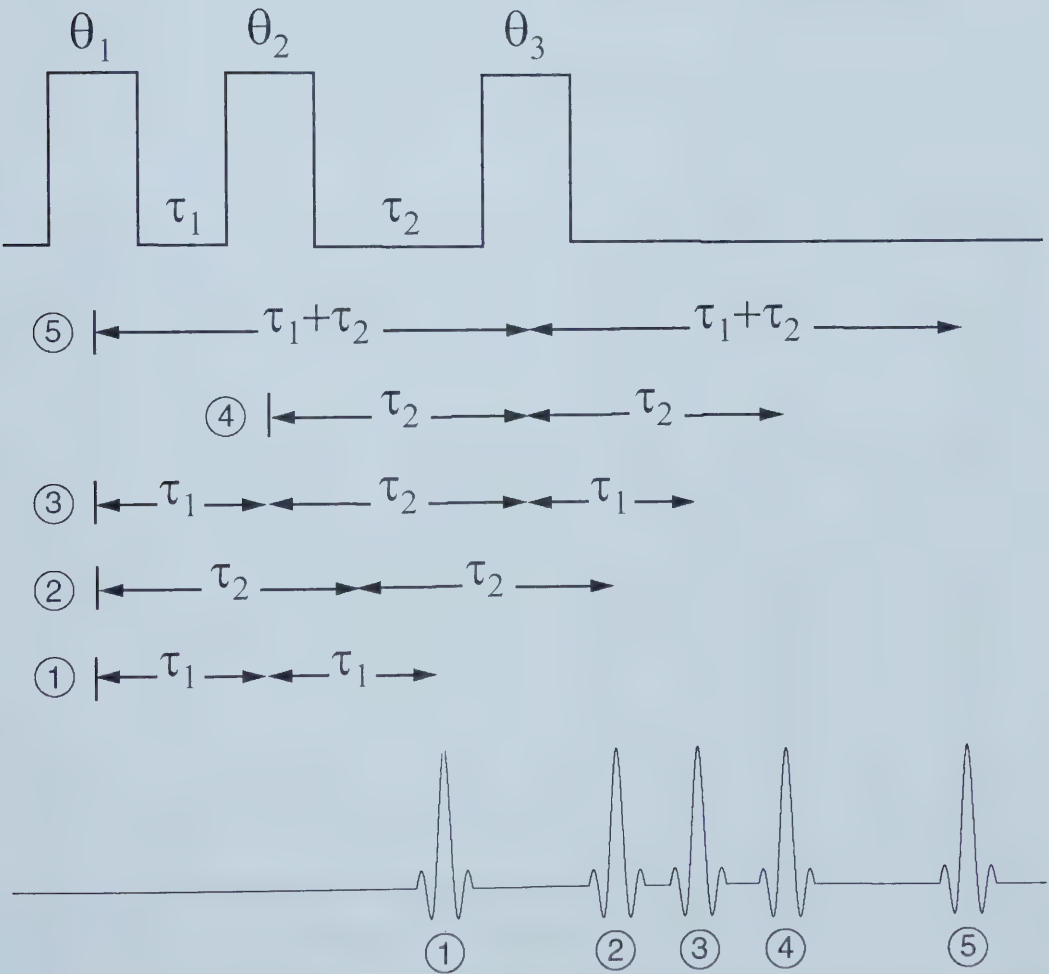


Figure 2.7: A NMR experiment with three RF pulses will generate a total of five echoes.

Table 2.1: Echo locations and peak amplitudes.

Echo	Time of Echo	Echo Amplitude
1	$2\tau_1$	$M_0 \sin(\theta_1) \sin^2\left(\frac{1}{2}\theta_2\right) e^{\frac{-2\tau_1}{T_2}}$
2	$2\tau_2$	$M_0 \sin(\theta_1) \sin^2\left(\frac{1}{2}\theta_2\right) \sin^2\left(\frac{1}{2}\theta_3\right) e^{\frac{-2\tau_2}{T_2}}$
3	$2\tau_1 + \tau_2$	$\frac{1}{2} M_0 \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) e^{\frac{-\tau_2}{T_1}} e^{\frac{-2\tau_1}{T_2}}$
4	$\tau_1 + 2\tau_2$	$M_0 \left\{ 1 - [1 - \cos(\theta_1)] e^{\frac{-\tau_1}{T_1}} \right\} \sin(\theta_2) \sin^2\left(\frac{1}{2}\theta_3\right) e^{\frac{-2\tau_2}{T_2}}$
5	$2(\tau_1 + \tau_2)$	$M_0 \sin(\theta_1) \cos^2\left(\frac{1}{2}\theta_2\right) \sin^2\left(\frac{1}{2}\theta_3\right) e^{\frac{-2(\tau_1 + \tau_2)}{T_2}}$

The trajectory of the magnetization vectors and the creation of stimulated echoes become increasingly more complex as the number of RF pulses increases. Thus, simulated echoes play an important role when imaging with FSE sequences (3, 6, 7, 18).

It is also desirable to have a method for predicting the total number, timing and magnitude of the stimulated echoes generated in a FSE sequence. Solving the Bloch equations to predict the location and amplitude of all the stimulated echoes is not practical or very intuitive. Extended phase graphs offer a simpler and more intuitive method of predicting echoes when multiple refocus pulses are used (10).

2.1.5 Extended Phase Graphs

To introduce extended phase graphs, a useful decomposition method originally described by Woessner will first be presented (25).

Consider a magnetization vector just after the application of a RF pulse with a flip angle of θ about the y' -axis:

$$M_{x'}^+ = M_{x'} \cos(\theta) - M_{z'} \sin(\theta) \quad [2.11a]$$

$$M_{y'}^+ = M_{y'} \quad [2.11b]$$

$$M_{z'}^+ = M_{x'} \sin(\theta) + M_{z'} \cos(\theta) \quad [2.11c]$$

Using the following complex conjugate notation:

$$M_{x'y'} = M_{x'} + iM_{y'} \quad [2.12a]$$

$$M_{x'y'}^* = M_{x'} - iM_{y'} \quad [2.12b]$$

the magnetization just after the RF pulse is:

$$M_{x'y'}^+ = M_{x'y'} \cos^2\left(\frac{\theta}{2}\right) - M_{x'y'}^* \sin^2\left(\frac{\theta}{2}\right) - M_{z'} \sin(\theta) \quad [2.13a]$$

$$M_{z'}^+ = M_{z'} \cos(\theta) + \frac{1}{2}(M_{x'y'} + M_{x'y'}^*) \sin(\theta) \quad [2.13b]$$

Notice that in the Woessner decomposition of Eq. [2.13] the transverse magnetization after the RF pulse has dependence on the transverse magnetization as well as the complex conjugate of the transverse magnetization just prior to the pulse (25). Consider the Woessner decomposition for a magnetization vector receiving either a π refocus pulse or no refocus pulse at all:

$$M_{x'y'} \xrightarrow{0} M_{x'y'} \quad [2.14a]$$

$$M_{x'y'} \xrightarrow{\pi_{y'}} -M_{x'y'}^* \quad [2.14b]$$

Thus, the fraction of the transverse magnetization that sees a π like pulse is $\sin^2 \frac{\theta}{2}$ and the fraction that is unchanged by the RF pulse is $\cos^2 \frac{\theta}{2}$. The portion of the transverse magnetization that receives a π like pulse will regain phase coherence and form an echo. In addition, if the flip angle is not exactly π , then Eq. [2.13] indicates that the longitudinal magnetization will contain a M_{xy}^* term. If the longitudinal magnetization is later flipped back into the transverse plane the M_{xy}^* term will regain phase coherence and form a stimulated echo.

An extended phase graph allows the formation of spin and stimulated echoes to be visualized by tracking the phase of magnetization components. In the laboratory frame the phases of the magnetization components are

$$\phi_{xy} = \phi_0 + \omega t \quad [2.15a]$$

$$\phi_z = \phi_0 \quad [2.15b]$$

such that any transverse component acquires phase linearly with time and any longitudinal component has no change in phase (Fig 2.8).

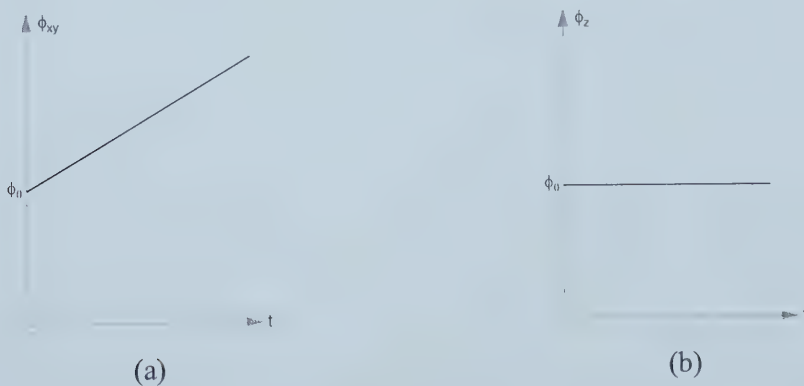
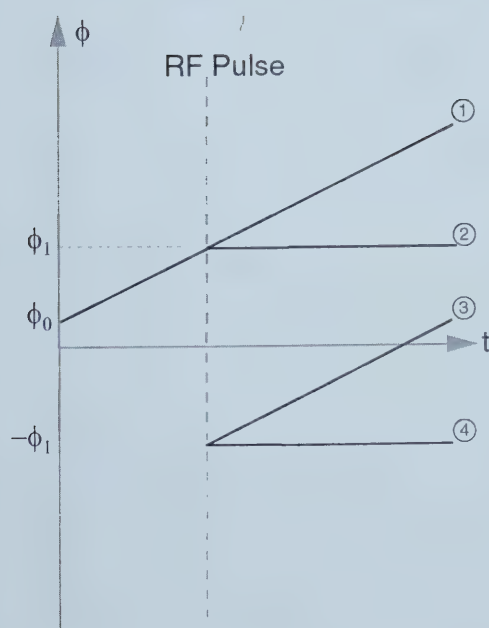


Figure 2.8: The free precession of the transverse magnetization in (a) creates a phase that is linearly dependent with time while in (b) the longitudinal magnetization has a constant phase.

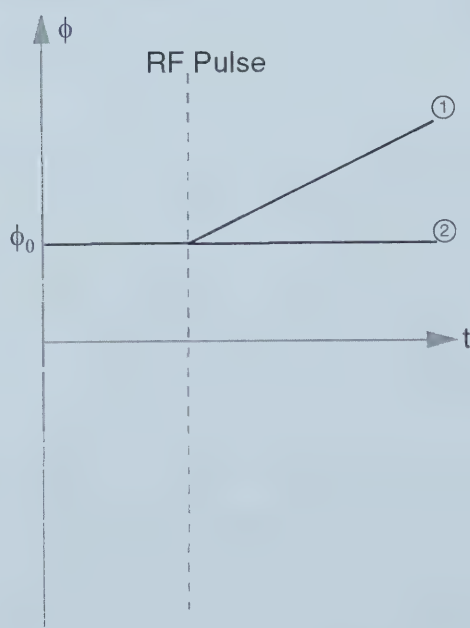
In an extended phase diagram a RF pulse is represented by a vertical line and each time the RF line intersects the phase line of a magnetization component, the phase line will branch into components governed by the branching rules. Each time a phase line crosses the time axis ($\phi = 0$) an echo is formed (Fig 2.9). The branching rules for an extended phase graph are:

$$M_{x'y'} \xrightarrow{\theta} \begin{cases} M_{x'y'}^+ = M_{x'y'} \cos^2\left(\frac{1}{2}\theta\right) \\ M_{x'y'}^+ = -M_{x'y'}^* \sin^2\left(\frac{1}{2}\theta\right) \\ M_{z'}^+ = \frac{1}{2} M_{x'y'} \sin(\theta) \\ M_{z'}^+ = \frac{1}{2} M_{x'y'}^* \sin(\theta) \end{cases} \quad [2.16a]$$

$$M_{z'} \xrightarrow{\theta} \begin{cases} M_{x'y'}^+ = -M_{z'} \sin(\theta) \\ M_{z'}^+ = M_{z'} \cos(\theta) \end{cases} \quad [2.16b]$$



(a)



(b)

Figure 2.9: Extended phase graphs for a transverse magnetization component (a) and a longitudinal component (b) demonstrate how the phase of each component is tracked. In (a), branch ③ crosses the time axis and will therefore produce an echo.

2.1.6 Maximum Number of Echoes

The concepts of the extended phase graph can be utilized to predict the total number of echoes generated by N_{RF} pulses. Let T_n be defined as the number of tilted branches in the phase diagram after the n^{th} pulse with H_n defined as the number of horizontal branches. Referring to Eq. [2.16], the number of branches after an additional RF pulse can be derived as:

$$T_{n+1} = 2T_n + H_n \quad [2.17a]$$

$$H_{n+1} = 2T_n + H_n \quad [2.17b]$$

Following the first excitation RF pulse there is one tilted branch ($T_n=1$) and one horizontal branch ($H_n=1$) possible. The recursion equations in Eq. [2.17] can be decoupled such that the number of tilted branches is solved as:

$$T_n = 3T_{n-1} \quad [2.18]$$

which has a solution:

$$T_n = 3^{n-1} \quad [2.19]$$

Only the tilted branches that are below the time axis will possibly evolve to form an echo. The number of echoes that can be generated after the n^{th} RF pulse is then:

$$N_E(n) = \frac{T_n - 1}{2} \quad [2.20]$$

To solve for the total number of echoes Eq. [2.20] must be summed over all the RF pulses:

$$N_{\text{echo}} = \sum_n N_E(n) \quad [2.21]$$

Given N_{RF} RF pulses, the number of echoes generated is then:

$$N_{\text{echo}} = \frac{3^{N_{\text{RF}}} - 2N_{\text{RF}} - 1}{4} \quad [2.22]$$

2.2 Spin Echo Imaging

2.2.1 Basic Spin Echo Sequence

MR Imaging methods can be classified as gradient echo, spin echo or a combination of the two. Gradient echo techniques have found wide spread use in clinical imaging especially with fast data acquisition and volume imaging applications. Nevertheless, gradient echo techniques suffer from restrictions set by susceptibility artifacts and magnetic field inhomogeneities. The improved T_2 contrast and ability to overcome the mentioned restrictions make spin echo imaging an attractive alternative.

Figure 2.10 illustrates a basic 2D Spin Echo imaging sequence. After the initial slice excitation and slice refocus, the phase and read gradients are applied to acquire a single phase encode line of k-space. The spin system is then allowed to return back to thermal equilibrium, for a period of time TR , before the segment is applied again for the next k-space line of data. If the image requires N_p pixels in the phase encode direction then the segment must be repeated N_p times to acquire the full image.

A three dimensional sequence is available by adding a set of phase encode gradients along the slice direction, however, the imaging time of a 3D spin echo sequence is a major barrier to routine clinical use. To gain a better idea of the imaging times involved, suppose that the repetition time needed in an experiment is one second ($TR = 1s$). A 2D spin echo MRI with 256×256 data pixels would have a total acquisition time of 4 minutes and 16 seconds. A $16 \times 256 \times 256$ 3D spin echo MRI would take 1 hour 8 minutes and 16 seconds, much too long to be applied clinically. The solution to reducing scan times is in the use of fast spin echo sequences.

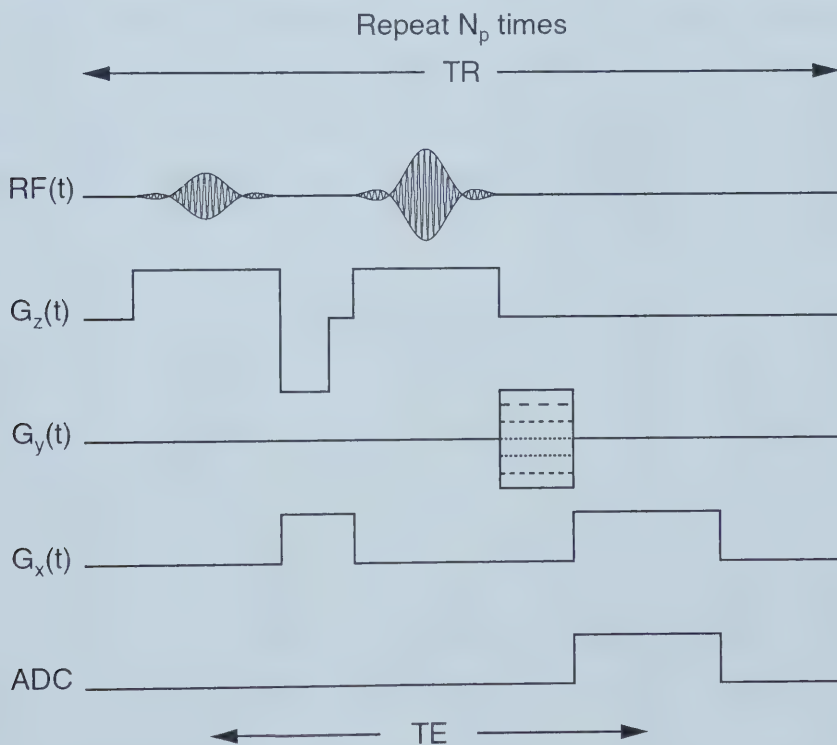


Figure 2.10: A basic 2D spin echo segment with an echo time of TE and a segment repetition time of TR , is repeated N_p times to acquire a full MRI. The sequence consists of a slice selective excitation followed by a read gradient lobe to set the starting position of the read data in k -space. The slice is refocused and a variable phase encode gradient is applied to set the starting position of the phase data in k -space. Finally the data is sampled with an analog to digital converter (ADC) while being frequency encoded by the read gradient. The spin echo will form at the center of the read gradient and will correspond to the central view of k -space in the read direction.

2.2.2 Basic Fast Spin Echo Sequence

Fast Spin Echo (FSE), Turbo Spin Echo (TSE) or Rapid Acquisition with Relaxation Enhancement (RARE) are techniques that acquire several lines of data after a single RF excitation as opposed to the single data view acquired with a spin echo sequence. Although every view needed for the image can be acquired after the first excitation, the data is more typically acquired in several passes or segments. If a FSE scan were to use 16 echoes per segment (N_E), the 16x256x256 3D MRI that took over an hour with a spin echo sequence would have a 16-fold improvement with a scan time of just over 4 minutes. The sequence shown in Fig. 2.11 is just one of many variations of FSE sequences (3, 6, 26). The specific design of a FSE sequence is dependent on the application of the MRI, physiological limitations and MR hardware limitations. The next chapters cover some applications and design considerations of the FSE sequence.

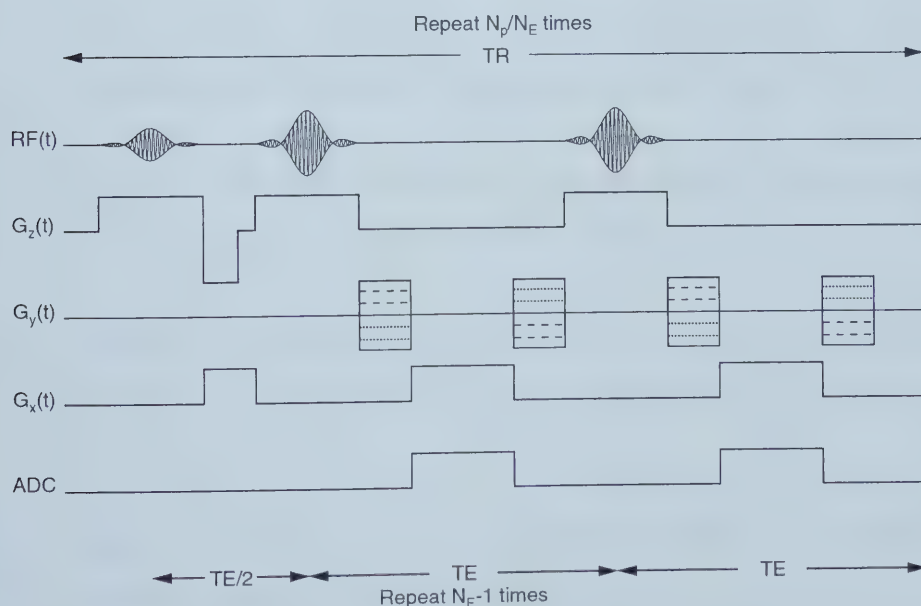


Figure 2.11: A FSE sequence utilizes the creation of multiple echoes to acquire several data views within a single segment.

2.3 Bibliography

1. Hahn EL. Spin Echoes. *Phys Rev* 1950; 80: 580-594.
2. Constable RT, Anderson AW, Zhong J and Gore JC. Factors Influencing Contrast in Fast Spin Echo MR Imaging. *Magn Reson Imaging* 1992; 10: 497-511.
3. Melki P, Mulkern RV, Dacher JN, Helenon O, Higuchi N, Oshio K, Einstein S, Jolesz F and Pourcelot L. Fast Spin-Echo MRI Techniques - Contrast Characteristics and Clinical Potential. *J Radiol* 1993; 74: 179-190.
4. Plewes DB and Bishop J. Spin-Echo MR Imaging. In: M. Bronskill and P. Sprawls. *The Physics of MRI - 1992 AAPM Summer School Proceedings*. Woodbury: American Institute of Physics, Inc.; 1993. 166-187
5. Hennig J and Friedburg H. Clinical Applications and Methodological Developments of the RARE Technique. *Magn Reson Imaging* 1988; 6: 391-395.
6. Hennig J, Nauerth A and Friedburg H. RARE Imaging: A Fast Imaging Method for Clinical MR. *Magn Reson Med* 1986; 3: 823-833.
7. Mulkern RV, Wong STS, Winalski C and Jolesz FA. Contrast Manipulation and Artifact Assessment of 2D and 3D RARE Sequences. *Magn Reson Imaging* 1990; 8: 557-566.
8. Oshio K and Jolesz FA. Fast MRI by Creating Multiple Spin Echoes in a CPMG Sequence. *Magn Reson Med* 1993; 30: 251-255.

9. Busse RF, Riederer SJ, Fletcher JG, Bharucha AE and Brandt KR. Interactive Fast Spin Echo Imaging. *Magn Reson Med* 2000; 44: 339-348.
10. Liang Z and Lauterbur PC. *Principles of Magnetic Resonance Imaging*. New York: IEEE Press; 2000. 416 pages.
11. Nishimura DG. *Magnetic Resonance Imaging*. Stanford: Stanford University; 1996. 223 pages.
12. Haacke EM, Brown RW, Thompson MR and Venkatesan R. *Magnetic Resonance Imaging - Physical Principles and Sequence Design*. New York: John Wiley & Sons, Inc.; 1999. 914 pages.
13. Bloch F. Nuclear Induction. *Phys Rev* 1946; 70: 460-474.
14. Purcell EM, Torrey HC and Pound RB. Resonance Absorption by Nuclear Magnetic Moments in a Solid. *Phys Rev* 1946; 69: 37-38.
15. Carr H and Purcell EM. Effects of Diffusion on Free Precession in Nuclear Magnetic Resonance Experiments. *Phys Rev* 1954; 94: 630.
16. Meiboom S and Gill D. Modified Spin-Echo Method for Measuring Nuclear Relaxation Times. *Rev Sci Instr* 1958; 29: 688.
17. Alsop DC. The Sensitivity of Low Flip Angle RARE Imaging. *Magn Reson Med* 1997; 37: 176-184.
18. Hennig J. Multiecho Imaging Sequences with Low Refocusing Flip Angles. *J Magn Res* 1988; 78: 397-407.

19. Debatin JF and McKinnon. Ultrafast MRI Techniques and Applications. New York: Springer-Verlag; 1998. 247 pages.
20. Reynolds HG and Rydberg J. Average RF Power Reduction for T2 FLAIR at 3.0 Tesla. Proceedings of the 8th Annual Meeting of ISMRM 2000; 3: p1988.
21. Mendes JK, DeZanche N, Emery DJ and Wilman AH. Whole Body Fast Spin Echo Imaging at Three Tesla. Proc ISMRM 2001; 2014.
22. LeRoux P and Hinks RS. Stabilization of Echo Amplitudes in FSE Sequences. Magn Reson Med 1993; 30: 183-190.
23. Hennig J and Scheffler K. Easy Improvement of Signal-to-Noise in RARE Sequences with Low Refocusing Flip Angles. Magn Reson Med 2000; 44: 983-985.
24. Hennig J. Echoes - How to Generate, Recognize, Use or Avoid them in MRI-Imaging Sequences Part I. Concepts in Magnetic Resonance 3 1991; 125-143.
25. Woessner DE. Effects of Diffusion in Nuclear Magnetic Resonance Spin-Echo Experiments. J Chem Phys 1961; 34: 2057-2061.
26. Fautz HP, Buchert M, Husstedt H, Laubenberger J and Hennig J. TSE Sequences with Spin Echo Contrast. Magn Reson Med 2000; 43: 577-582.

3

Fast Spin Echo Sequence Design

Magnetic Resonance Imaging techniques based on fast spin echo sequences have become a routine part of clinical imaging (1-4). The FSE sequence utilized in this thesis was developed on a 3.0 T whole body system (Magnex magnet). The system utilizes a console developed by Surrey Medical Imaging Systems (SMIS) and has been in operation since 1994. The scanner is equipped with a gradient set capable of 20mT/m with a 100mT/m/ms slew rate. The system is equipped with an array of RF coils including a 56cm diameter transmit-receive whole body quadrature coil (developed in a research collaboration with Marconi Medical Systems) and a quadrature transmit-receive elliptical neck coil with major and minor axes of 30cm and 23cm respectively (Medical Advances, Milwaukee, USA). The RF coils are powered with an 8kW RF amplifier (AMT-3T80, American Microwave Technology, Anaheim, CA).

Programming the SMIS console is accomplished with a pulse programming language (PPL). This chapter discusses the original development and application of the PPL code for a FSE sequence running on the above-described MR system.

3.1 Radio Frequency Design

3.1.1 Specific Absorption Ratio

The average RF power per unit volume deposited into a sample is given by Eq. [1.48]. To find the total power deposited into a sphere of radius R , Eq. [1.48] must be integrated over the volume of the sphere:

$$P = \int_{\text{volume}} \langle P \rangle dV \quad [3.1]$$

To a first order approximation the total RF power deposition in a sphere is:

$$P = \frac{\sigma \pi \omega_{\text{rf}}^2 B_1^2 R^5}{15} \quad [3.2]$$

where R is the radius of the sphere being considered and ω_{rf} is the angular frequency of the B_1 field (5). The specific absorption rate (SAR) is defined as the energy per unit volume per unit time that is delivered to a sample. Using the same spherical sample from Eq. [3.2]:

$$\text{SAR}_{\text{AVG}} = \frac{\sigma D \omega_{\text{rf}}^2 B_1^2 R^2}{20\rho} \quad [3.3]$$

where σ is the conductivity of the sample, ρ is the density of the sample and D is the radio frequency duty cycle (6). The SAR is a critical factor in determining the amount of sample heating that will occur as a result of a MRI sequence. Governing agencies such as the FDA Center for Devices and Radiological Health place strict standards on the SAR allowed in MR sequences (7).

At the time this thesis was written, the current FDA guidelines prevent the use of a MRI sequence that has a specific absorption rate greater than:

- a) 4 W/kg averaged over the whole body for any period of 15 minutes; or
- b) 3 W/kg averaged over the head for any period of 10 minutes; or
- c) 8 W/kg in any gram of tissue in the head or torso, or 12 W/kg in any gram of tissue in the extremities, for any period of 5 minutes;

The SAR relation presented in Eq. [3.3] is an approximation of the average SAR over the entire sphere. However, the B_1 field distribution is not spatially uniform, especially at higher magnetic field strengths (8-17). This will result in a power deposition that is also spatially dependent (18-20). Figure 3.1 demonstrates that although the average SAR might not exceed the FDA limits, it is possible that the local SAR limit specified in part (c) of the FDA guidelines will be violated.

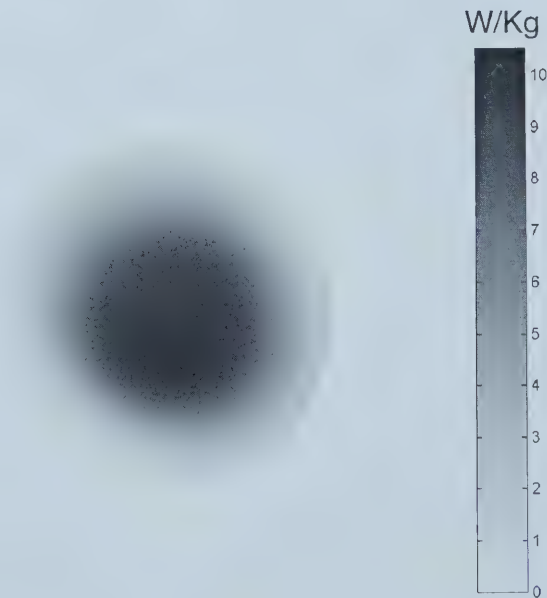


Figure 3.1: The local SAR distribution at 3.0 T of a cylindrical phantom (3L) was based on an 8-echo FSE sequence with a repetition time of 1 second. The 90° excitation pulse had an average power of 80W and each 180° refocus pulse had an average power of 348W. The average SAR was 1.9 W/Kg.

Regardless of whether average or local SAR limits are exceeded, an alternative RF design will need to be implemented for the MR sequence to be clinically viable (20-23). There is also increased power deposition at higher magnetic field strengths due to the ω_{rf}^2 dependence of SAR. This places even stricter boundaries on the use of RF intensive sequences (24-26). The limits imposed by SAR will therefore be a significant factor in designing RF pulse envelopes and determining the flip angles used in a MR sequence.

3.1.2 Excitation Pulse Envelope

In the first chapter, the concept of slice selection and the Fourier transform method of predicting slice profiles were introduced. This method of predicting the slice profile employs the spectrum of a RF pulse envelope to predict the resulting distribution of flip angles across a slice. Ideally the slice profile should be a boxcar shape with perfect excitation within the slice and no excitation outside the slice. The pulse envelope required for a boxcar slice profile is an infinitely long Sinc pulse.

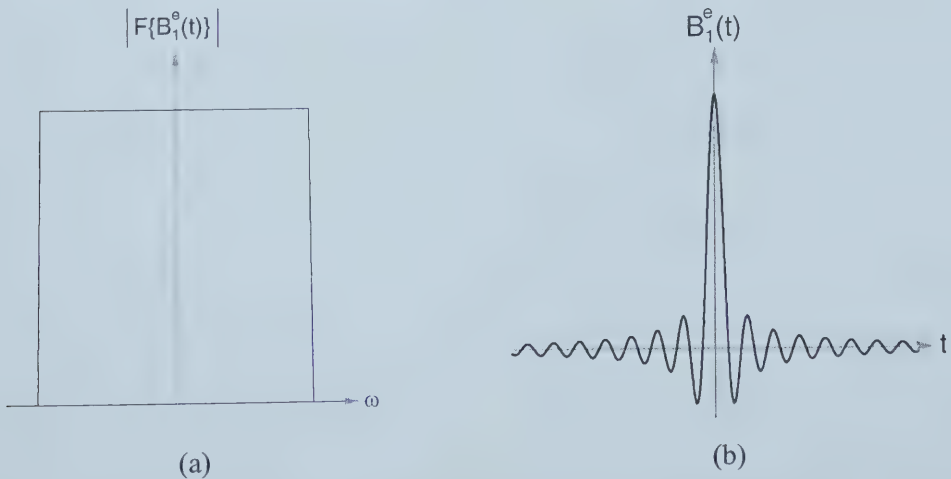


Figure 3.2: The Fourier transform method predicts that a boxcar slice profile requires a pulse envelope with a boxcar spectrum (a). The RF pulse envelope required for a boxcar spectrum is an infinitely long Sinc function, shown truncated in (b).

The time scale of an RF pulse in most MR Imaging experiments is typically on the order of a few milliseconds. A decision must therefore be made as to where the infinite Sinc function should be truncated. The Sinc function is typically truncated at a point where the function is zero and the new RF envelope is identified by the number of lobes remaining in the Sinc function.

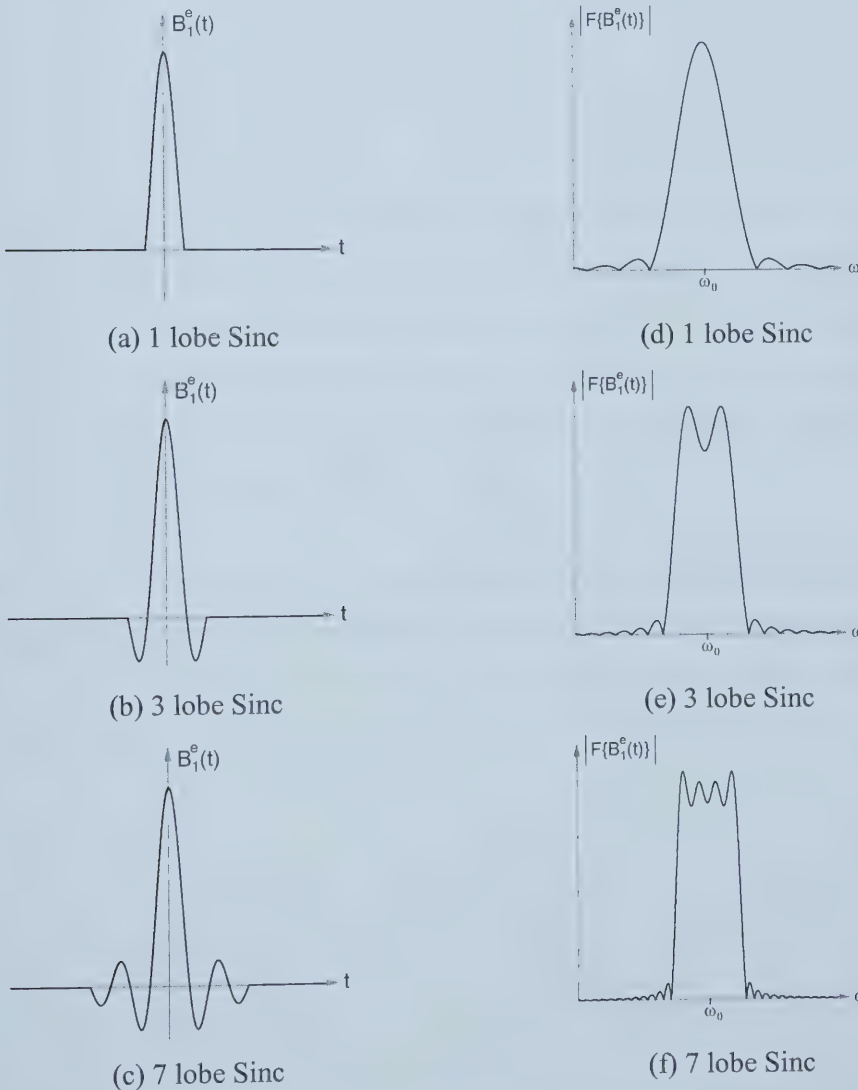


Figure 3.3: The truncated Sinc pulse envelopes in (a)-(c) produce the spectra shown in (d)-(f) respectively.

Truncating the Sinc function will consequently affect the spectrum of the RF envelope and a compromise between pulse length and slice profile will need to be made (27). As the number of lobes in the pulse envelope decreases, the spectrum becomes less like a boxcar and more like a Sinc function. From the point of view of the slice profile, it is desirable to maintain as many of the side lobes as possible. However, the flip angle of a RF pulse is defined as:

$$\theta = \gamma \int_{\tau_p} B_1^e(t) dt \quad [3.4]$$

For a given pulse length τ_p and flip angle θ , retaining more side lobes will require a larger peak value of $B_1^e(t)$. Equation [3.3] stated that SAR has a B_1^2 dependency on the RF envelope. The three-lobe Sinc envelope has half the power deposition as an equivalent flip angle seven-lobe Sinc envelope. The result is that although more side lobes can improve the slice profile, the penalty of increased SAR might be a more important factor in selecting a RF pulse envelope.

There is also a wide variation of filtering techniques that can further improve the slice profile and tailor the pulse to a specific application without significantly affecting SAR (28-30). For this thesis, a Kaiser filtered has been selected because it offers good suppression of the ringing effect caused by the truncation of the Sinc function.

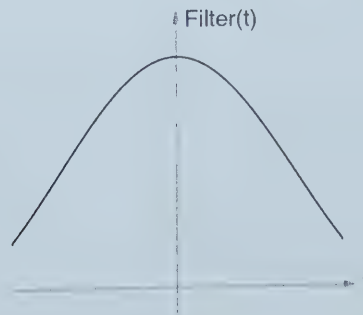


Figure 3.4: General shape of a Kaiser filter.

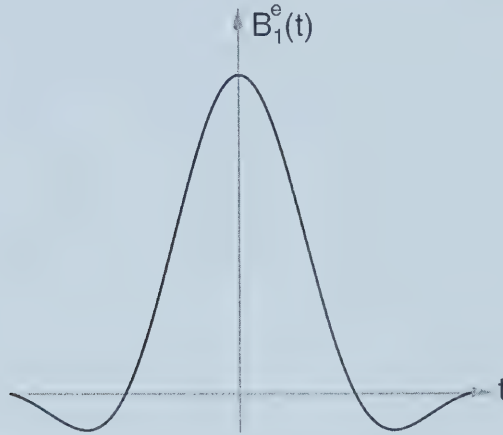


Figure 3.5: The Kaiser filtered three-lobe Sinc function used as the pulse envelope.

3.1.3 Excitation Profile

The excitation profile is defined as the magnitude of the transverse magnetization vectors across a slice after the application of an on resonance excitation pulse to a system at thermal equilibrium. An excitation flip angle of $\pi/2$ is large enough such that the Fourier transform method does not provide an accurate slice profile and a numerical solution to the differential Bloch equation must be solved instead. For a three-lobe Kaiser filter Sinc envelope, the slice width is:

$$\Delta z = \frac{3.8}{\gamma G_{ss} \tau_p} \quad [3.5]$$

where G_{ss} is the slice select gradient strength in T/m and τ_p is the length in seconds of the RF pulse. This relation was estimated using numerical simulations and verified experimentally. Figure 3.6 shows the typical slice profile that can be expected when a Kaiser filtered three-lobe Sinc RF excitation pulse is applied.

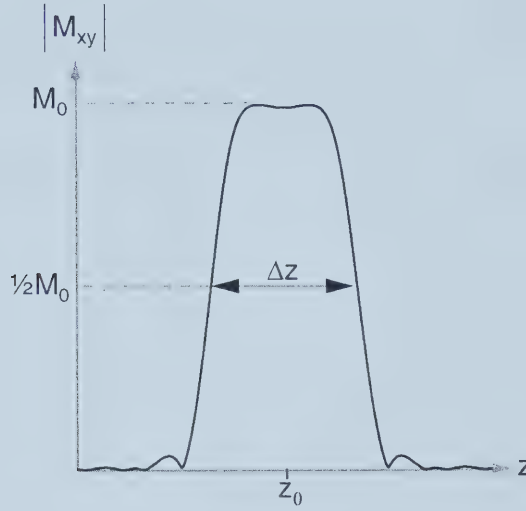


Figure 3.6: The slice excitation profile with a 3-lobe Kaiser filtered RF pulse.

3.1.4 Refocus Pulse Envelope

The ideal RF refocus pulse is able to uniformly transform the phase of every transverse magnetization component without changing the magnitude or exciting any longitudinal component into the transverse plane. For example, an ideal refocus pulse applied along the y-axis should have the following effect:

$$M_{x'y'} \xrightarrow{\pi_{y'}} -M_{x'y'}^* \quad [3.6]$$

In some spin echo applications it is advantageous to have a refocus pulse envelope that differs from the excitation pulse envelope (31). However, in general it will suffice to use the same envelope as the excitation pulse and the spin echo sequences used in this thesis will follow this trend. Figure 3.7 shows the distribution of $M_{x'}$ after the application of a π_y refocus pulse to a system of spins uniformly excited along the x' -axis.

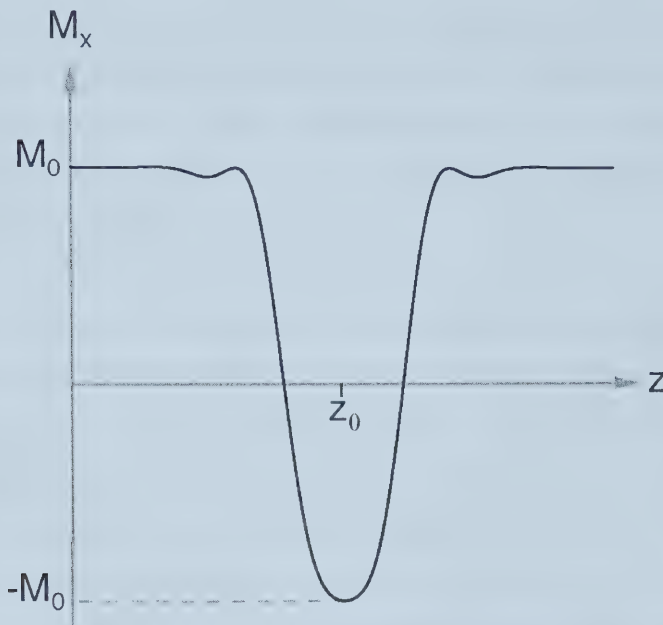


Figure 3.7: The final x' -components of a distribution of equal magnitude transverse magnetization, initially along the x' -axis, after being subjected to a π_y refocus pulse.

3.1.5 Refocus Pulse Phase

In the second chapter, the effect of a perfect π -refocus pulse on the transverse magnetization was investigated. Equation [2.9] indicated that any reversible phase distribution could be fully eliminated regardless of the phase of the RF pulse. It would stand to reason that, as long as the refocus pulse flip angles remain as near to 180° as possible, the phase of each refocus pulse in a FSE sequence should not cause any additional loss of signal. However, in practice, careless selection of the phase of each refocus pulse will lead to an almost complete loss of signal along an echo train.

Equation [2.22] predicted that the number of echoes generated by a FSE sequence was not always equal to the number of refocus pulses. For reduced refocus flip angles the creation of stimulated echoes after the application of a second RF pulse increases the

expected number of echoes generated. In addition, the phase of the stimulated echoes are not necessarily the same and the echoes can destructively interfere with each other to produce a net loss of signal. Since stimulated echoes are not created until after the application of a second RF pulse, this problem does not exist in spin echo sequences or with the first echo of a FSE sequence.

Once again it would stand to reason that if near 180° refocus pulse flip angles were used to minimize stimulated echoes then the problem would be solved. However, this solution can never be realized in a NMR experiment. It has already been shown that the distribution of flip angles will vary in the slice direction due to non-ideal excitation pulses (§3.1.4). In a 2D scan, the refocus flip angles across a voxel in the slice direction will range from the maximum refocus angle all the way to zero. In a 3D scan, the voxels near the edge of the excited slab will suffer from a similar distribution of reduced refocus flip angles.

Due to this distribution of reduced refocus flip angles across the slice there will always be the creation of stimulated echoes after the second RF refocus pulse. It is therefore important to plan the phase of the refocus pulses such that the stimulated echoes will constructively contribute to the signal rather than destructively contribute to signal loss (2, 3, 32, 33). Although not totally immune to the effects of signal loss due to reduced flip angles, a CPMG RF configuration offers significantly improved performance over many other configurations and is the arrangement used in this thesis unless specifically mentioned otherwise (34, 35).

3.1.6 Experimental Phase Errors

On a MR scanner, the phase of a RF refocus pulse is controlled through the phase of the carrier wave produced by a synthesizer. If the frequency of the synthesizer is set to

exactly match the precessional frequency of the spins the two systems will remain in phase throughout the experiment. However, due to magnetic field inhomogeneities and errors in the synthesizer hardware, it will be impossible to exactly match the frequencies and some phase errors between the two systems will arise. The previous section discussed the importance of the phase of a refocus RF pulse and these phase errors could result in the eventual loss of signal. Fortunately, these frequencies can typically be matched close enough such that any signal loss due to this particular phase error is minimized.

There is another possible source of phase error that occurs in a MR experiment. This phase error arises when the phase of the spins are manipulated with a gradient. If the synthesizer is unaware of these phase changes then the next application of a RF pulse will result in the pulse being applied at the wrong phase.

To illustrate this point, consider the process of slice selective excitation. During slice selection, a gradient and RF pulse are applied simultaneously to excite a slice of spins. The thickness of the slice depends on the RF bandwidth and the location of the slice depends on the carrier frequency of the RF pulse. As a result, during the RF pulse the synthesizer frequency is temporarily adjusted to allow control over the slice location. After the RF pulse has completed, the synthesizer is returned to the resonant frequency and the slice gradient is turned off. Both systems will still be in phase since the frequency change of the synthesizer was also experienced by the spins due to the slice gradient. However, an additional slice rephase gradient is then applied to remove the phase distribution acquired by the spins during excitation. The synthesizer is not aware of this phase change and will therefore become out of phase with the spin system (Fig 3.8). This phase error can lead to a considerable loss in signal and should therefore be accounted for by offsetting the phase of each successive refocus pulse.

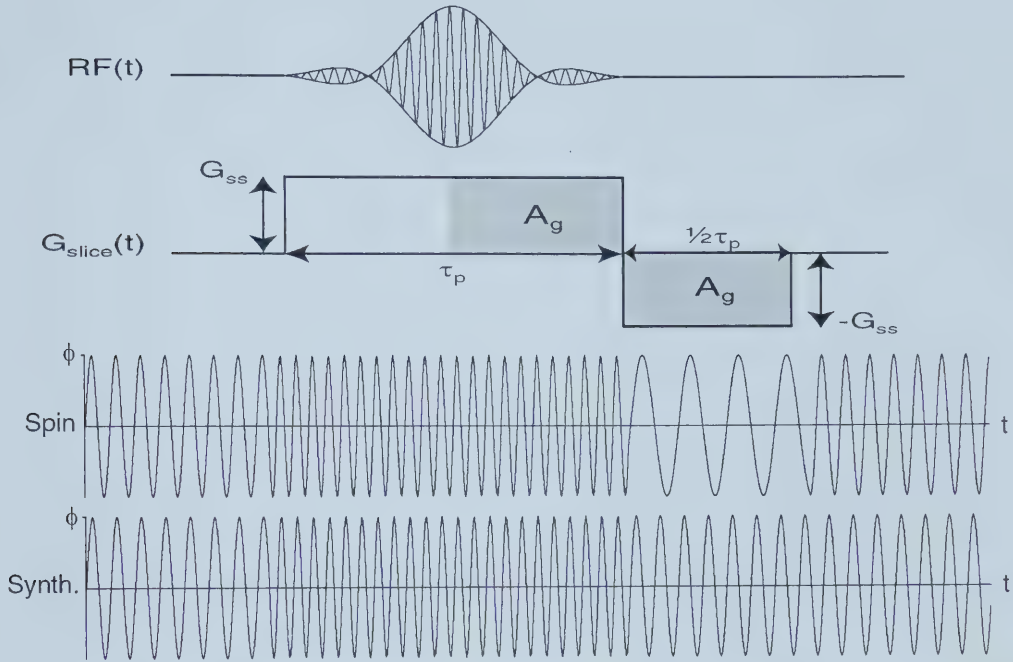


Figure 3.8: Demonstration of how the slice select process can create a phase difference between the spin system and the synthesizer. During excitation both the spin precessional and synthesizer frequency are identical and no phase errors occur. When the spins are rephased with a negative gradient, the phase change is invisible to the synthesizer and the two systems become out of phase.

The FSE sequence is therefore designed with a slice phase correction defined as:

$$\phi_{\text{slice_correct}} = -\pi\gamma G_{\text{ss}}\tau_p z_0 \quad [3.7]$$

where G_{ss} is the slice select gradient strength, τ_p is the length of the RF pulse and z_0 is the location of the slice. Since the phase correction depends on slice location it must be calculated for each individual slice in a multi slice experiment. This phase offset has been experimentally verified to restore the echo amplitudes (Fig. 3.9).

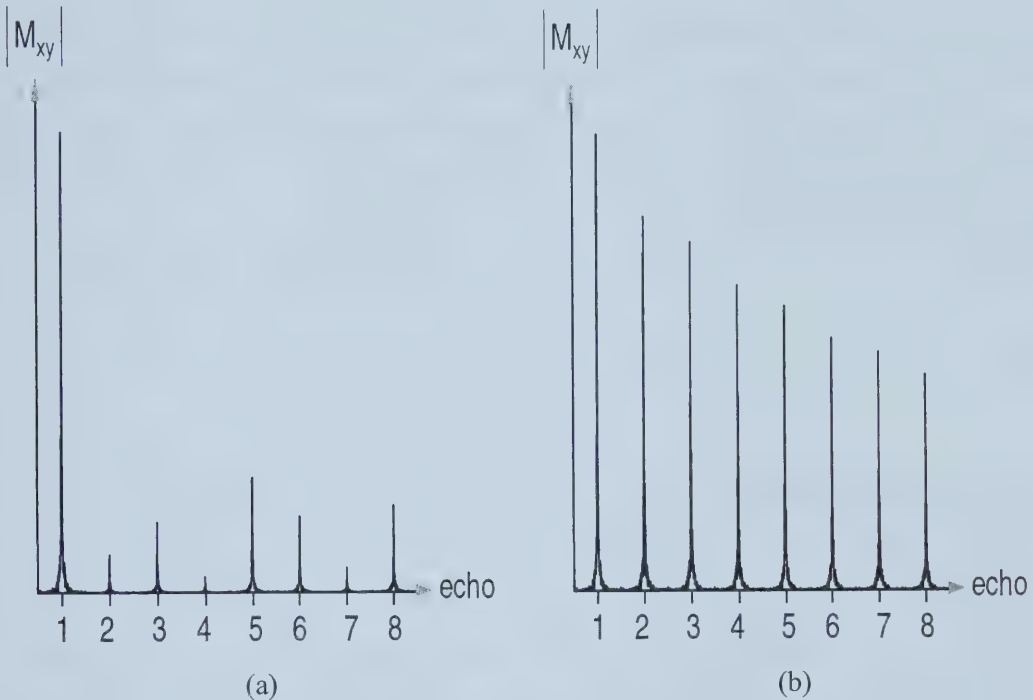


Figure 3.9: The loss of signal amplitude due to a slice offset phase error is clearly observable in (a). The application of the phase correction given in Eq. [3.7] restores the echo amplitudes to the expected values.

The slice phase error is caused by a phase change in the spin system while the synthesizer is not aware of the change. A similar phase error can occur when the synthesizer changes phase and does not communicate this change to the spin system. Such a problem arises when the image is offset in the read direction.

The precession frequency of a spin during data acquisition is spatially dependent due to the application of a read gradient. This location of a spin is then calculated by analyzing the spectrum of the sampled data signal. The spin can artificially be offset in the read direction by shifting the entire spectrum of the sampled signal. The FOV of a MR image can therefore be offset in the read direction by frequency shifting the data signal before it is sampled (36, 37).

On the 3.0 T system used in this thesis, there is one synthesizer common to both the receive and transmit RF systems. If the FOV is shifted in the read direction then the synthesizer frequency is modified during data acquisition. The spin system is not aware that the image FOV has been shifted and the resulting synthesizer frequency change creates a phase error between the two systems. Once again the phase error can be corrected with the application of an offset phase:

$$\phi_{\text{read_correct}} = 2\pi\gamma\tau_{\text{acq}}G_{\text{read}}\Delta r \quad [3.8]$$

where τ_{acq} is the total acquisition time, G_{read} is the strength of the read gradient and Δr is the FOV offset in the read direction. It is important to note that Eqs. [3.7] and [3.8] describe unexpected phase changes directly resulting from the experimental implementation of the MR pulse sequence. There are many instances when a phase correction is needed due to the NMR physics of the sequence itself. For example, Eq. [2.9] indicated that with a π -refocus pulse the final phase of a spin would differ from the initial phase depending on the phase of the RF refocus pulse. This phase change is a result of the NMR sequence itself and not of the implementation of the sequence.

3.1.7 Experimental Investigation of Duty Cycle

Equation [3.3] indicates that in addition to sample properties and RF pulse design, the average power deposition is also dependent on the duty cycle of the RF pulse. On a MR system, the average RF amplifier power output is checked every few seconds with a RF monitor to verify that SAR guidelines are being met. In a spin echo sequence the duty cycle is controlled by the echo time (time between RF pulses in a segment) and the repetition time (time between segments). The length of a spin echo segment is typically much less than the averaging time period used by the RF monitor and only the repetition time between segments need be considered. The limits placed on the RF duty cycle by

SAR guidelines were investigated for both a neck and body coil at 3.0 T (38). The peak power required for the 90° and 180° RF pulses was measured for each coil. For the neck coil the neck images required an average of 250 Watts for the 90° pulse and 1 kW for the 180° pulse. Both of these pulses were 2ms in length and modulated by a three-lobe Sinc function. In the case of the body coil, this peak power jumped to 1.4kW and 5.5kW for 7ms, three-lobe Sinc modulated 90° and 180° pulses respectively.



Figure 3.10: Coronal 256x256 FSE body image obtained while experimentally examining the limits of the RF duty cycle. The image was acquired with an echo train length of 6 echoes and a echo train spacing of 17 ms.

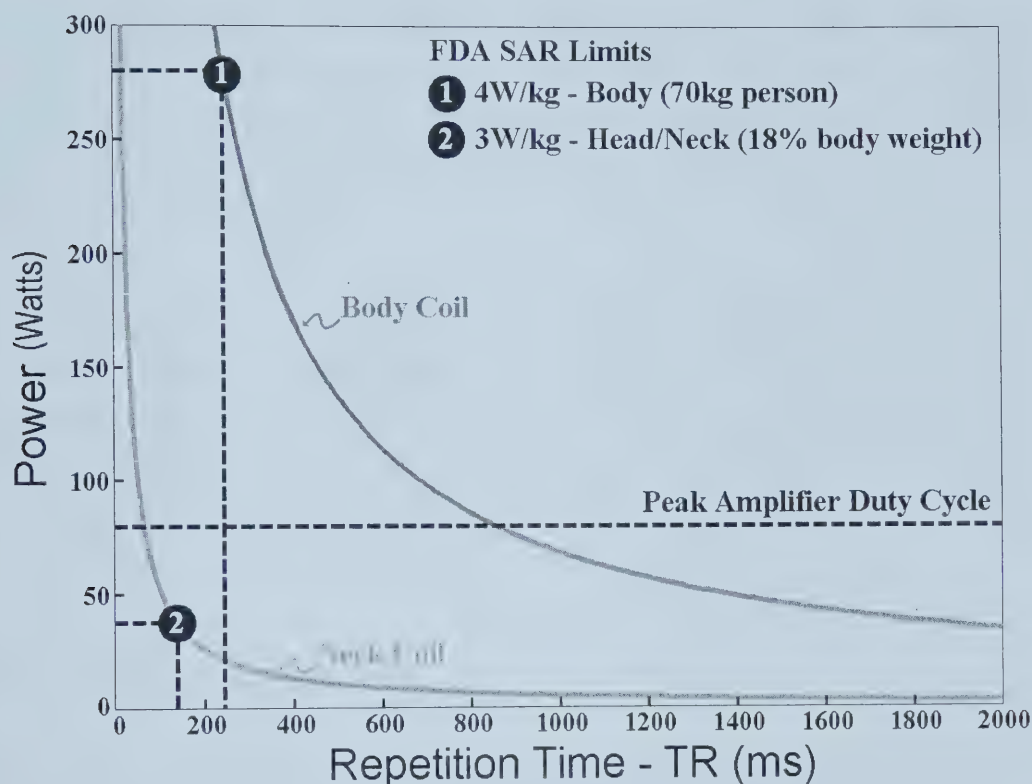


Figure 3.11: Patient absorbed power given a desired repetition time. The power calculation assumes a complete segment is acquired each TR period. A segment consists of the initial 90° excitation pulse followed by an echo train of refocus pulses.

For the neck coil, the limiting factor is the 3W/kg SAR cut-off which limits the minimum TR time to ~120ms. This is the minimum TR allowed by SAR limits and there are other factors, such as echo train length, that will also place limits on the minimum TR allowed. If SAR is the limiting factor, a 256x256 image with an ETL of 8 corresponds to about 15 slices per minute. With the body coil, it was observed that the limiting factor was the peak duty cycle available from the RF amplifier. Although the amplifier has a peak power of 8kW, it can only sustain an 80 Watt average output power.

The SAR cut-off (4W/kg) for the body coil allowed for a TR of ~250ms, however, the limited amplifier average power reduced this to about 900ms (3 slices per minute). Utilizing 160° pulses instead of 180°, the minimum TR shrinks to 700ms (4 slices/min), 140° giving 550ms (5 slices/min) and finally a 120° pulse would result in a minimum TR of 400ms (7 slices/min).

As whole body RF coils become more efficient and higher power RF amplifiers become available, the duty cycle limit of body imaging will also reach the restrictions imposed by SAR guidelines. Since the repetition time is a critical factor in both image acquisition time as well as RF duty cycle, SAR guidelines will ultimately place an upper limit on the speed of image acquisition.

3.1.8 Fat Saturation Pulse

When atoms are placed in a static magnetic field $\mathbf{B}_0$ their electrons will circulate causing a small additional magnetic field. In most cases this small magnetic field will oppose $\mathbf{B}_0$ and the electrons effectively shield the nucleus from the main static magnetic field. Since the resonant frequency of a nucleus is dependent on the net magnetic field, this shielding of $\mathbf{B}_0$ will result in a lowered resonant frequency. This phenomenon is called chemical shift and is measured using a ppm (parts per million) ratio:

$$\delta = \frac{\omega - \omega_{\text{ref}}}{\omega_{\text{ref}}} 10^6 \quad [3.9]$$

where δ is the field independent scalar ratio (in ppm) between the resonant frequency of interest ω and a reference frequency ω_{ref} (39). The hydrogen nuclei in water are bound to a single oxygen atom while the hydrogen nuclei in fat are bound to a large chain of carbon atoms.

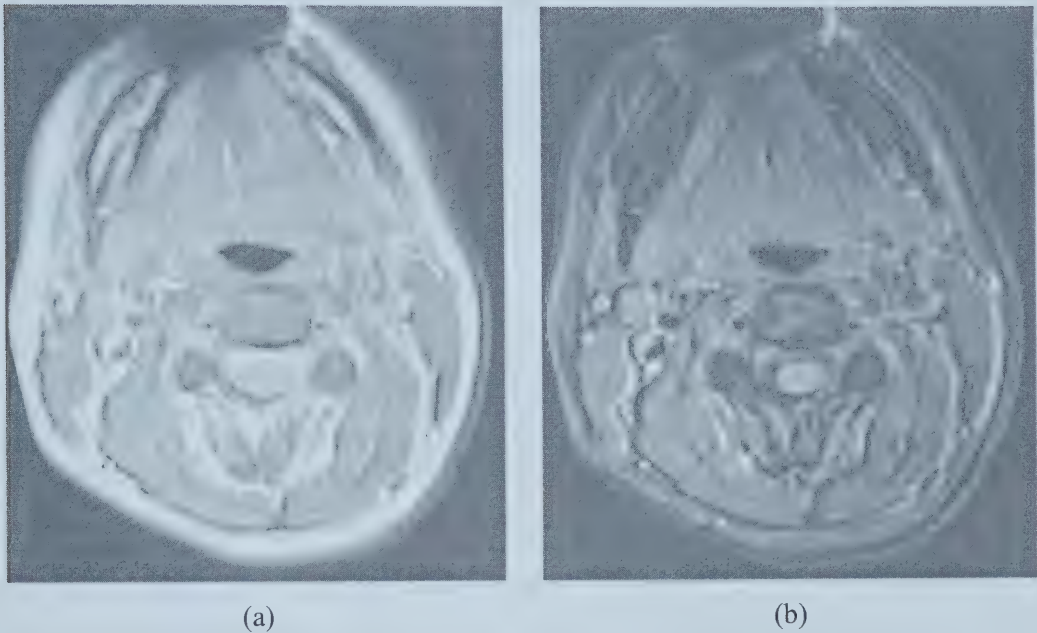


Figure 3.12: A FSE 3.0 T sagittal neck image shown with no fat suppression (a) and with a spectrally selective fat suppression (FS) pulse applied. The fat saturation pulse is applied just before each 8-echo segment. The images are 256x256 with a 250 mm FOV and a 5 mm slice thickness. The repetition time is 2000 ms and the echo spacing and echo times were both 9 ms.

This leads to a difference in magnetic environments and a corresponding chemical shift of ~ 3.5 ppm. At 3.0 Tesla the resonant frequency of hydrogen nuclei in lipids resonate approximately 440 Hertz below the resonant frequency of hydrogen in water. This frequency offset is small enough that both lipid and hydrogen nuclei will be excited during slice selection, although the lipids are excited from a slice offset from the water slice. During frequency encoding this frequency difference will lead to a displacement in the read direction of the fat in an image. This is one of the reasons why a wide array of MR techniques has been designed to reduce or remove the fat signal from an image (40-42).

The technique of spectral saturation has been selected for this FSE sequence. This method of fat suppression requires just one additional RF pulse, applied with the appropriate frequency offset, prior to the imaging block of the sequence. Figure 3.12 demonstrates the use of a three-lobe Kaiser filtered pulse, applied at the start of each segment, to spectrally suppress the fat signal from an image.

3.1.9 Magnetization Transfer Pulse

The water molecules in tissue can be classified as either freely moving or as being bound to a macromolecule. The relatively short T_2 relaxation times of bound hydrogen ($<1\text{ms}$) result in a large bandwidth ($>15000\text{Hz}$) power spectrum. This power spectrum is centered on the much narrower bandwidth ($<10\text{Hz}$) power spectrum of the freely moving hydrogen (Fig. 3.13).

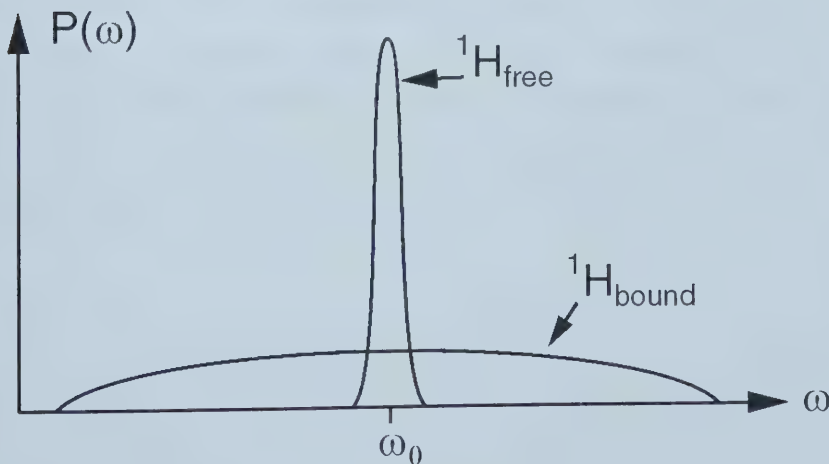


Figure 3.13: Power spectrum of bound and free hydrogen nuclei.

The wide bandwidth of the bound hydrogen allow them to be excited at a frequency far enough off resonance that the free hydrogen are only minimally excited. The signal from the bound protons is short lived and typically decays away before any data can be acquired. However, these bound hydrogen molecules will exchange energy with the free hydrogen molecules. The loss of magnetization of the bound hydrogen molecules and the excitation of magnetization for the free hydrogen is termed magnetization transfer (MT). The irradiated bound hydrogen nuclei will consequently decrease the steady state longitudinal magnetization of the free hydrogen nuclei (43). Thus magnetization transfer is used in MR imaging to provide increased contrast between tissues that contain macromolecules and those that do not.

One of the drawbacks of magnetization transfer is the large power deposition required to fully saturate the macromolecule containing tissues. The FSE sequence is already a RF intensive sequence and staying within SAR limits is an increasing challenge at higher field strengths. As a result, the use of magnetization transfer in FSE sequences will generally require a decrease in the MT pulse power and a corresponding reduction in the contrast available. To help minimize the power deposition of the MT pulse, a Gaussian pulse envelope is selected instead of the standard three-lobe Sinc. The Gaussian envelope has a lower average B_1 compared to an equivalent flip angle three-lobe Sinc, but at the cost of an inferior frequency profile. Figure 3.14 demonstrates the reduction of signal from tissue containing macromolecules with the use of MT.

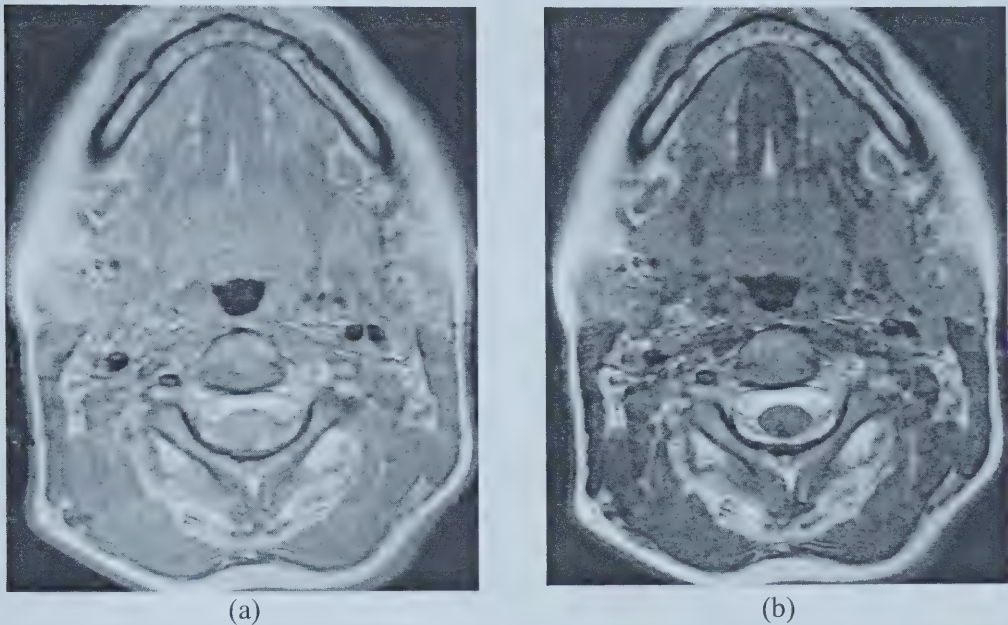


Figure 3.14: A FSE 3.0 T sagittal neck image without MT (a) and with a magnetization transfer preparation pulse (b). Tissues such as muscle have a visible signal drop with MT but CSF, which does not contain macromolecules, has a minimal change in signal. The MT pulse had a Gaussian envelope, a pulse length of 30ms and an offset frequency of 1000Hz. The images are 256x256 with a 180 mm FOV and a 5 mm slice thickness. The repetition time is 2000 ms and the echo spacing and echo times were both 10 ms.

3.2 Gradient Design

3.2.1 Ramp Time

Up to this point it has been assumed that a gradient magnetic field could be instantaneously turned off and on. A gradient coil is constructed by wrapping a length of wire into an appropriate configuration (44). As a result, a gradient coil will have a non-negligible inductance L . Using the voltage-current relation:

$$V = L \frac{dI}{dt} \quad [3.10]$$

It is seen that an instantaneous change in current would require an infinite driving voltage. Consequently, a gradient will always require a finite period of time to turn on and off. This time period is referred to as the ramp or rise time of the gradient coil. The maximum rate at which the gradient strength can be changed is called the slew rate of the gradient coil. For the 3.0 T MR system used in this thesis, the gradients have a maximum strength of 20mT/m and a slew rate of 100mT/m/ms resulting in maximum rise time of 200 μ s. This rise time must be taken into consideration when designing the gradients for a MR sequence.

The switching of gradients on and off produces a time varying magnetic field that will induce a current in a nearby conductor. In particular, a switching gradient field has the ability to stimulate peripheral nerves of patients undergoing an MR exam (45, 46). The FDA prevents the use of gradients with a time rate of change (dB/dt) sufficient to produce severe discomfort or painful nerve stimulation (7). One study has found the threshold for EMG activity for a rise time of 300 μ s to be about 20 mT/m (47). The slew rate required for painful stimulation is still being researched but experimental use of a 200 μ s rise time on the 3.0 T MR scanner has not lead to any significant peripheral nerve stimulation.

Even if the rapid switching does not cause peripheral nerve stimulation, as the gradient coils switch on and off they will experience a torque due to the main static magnetic field. This changing torque causes the coils to rapidly vibrate producing sound noise. The FDA prevents the use of a MR sequence that produces peak un-weighted sound pressure level greater than 140 dB or A-weighted r.m.s. sound pressure level greater than 99 dBA with hearing protection in place (7). The ramp times of a sequence should therefore be optimized to balance patient comfort and sequence performance.

3.2.2 Slice Gradient

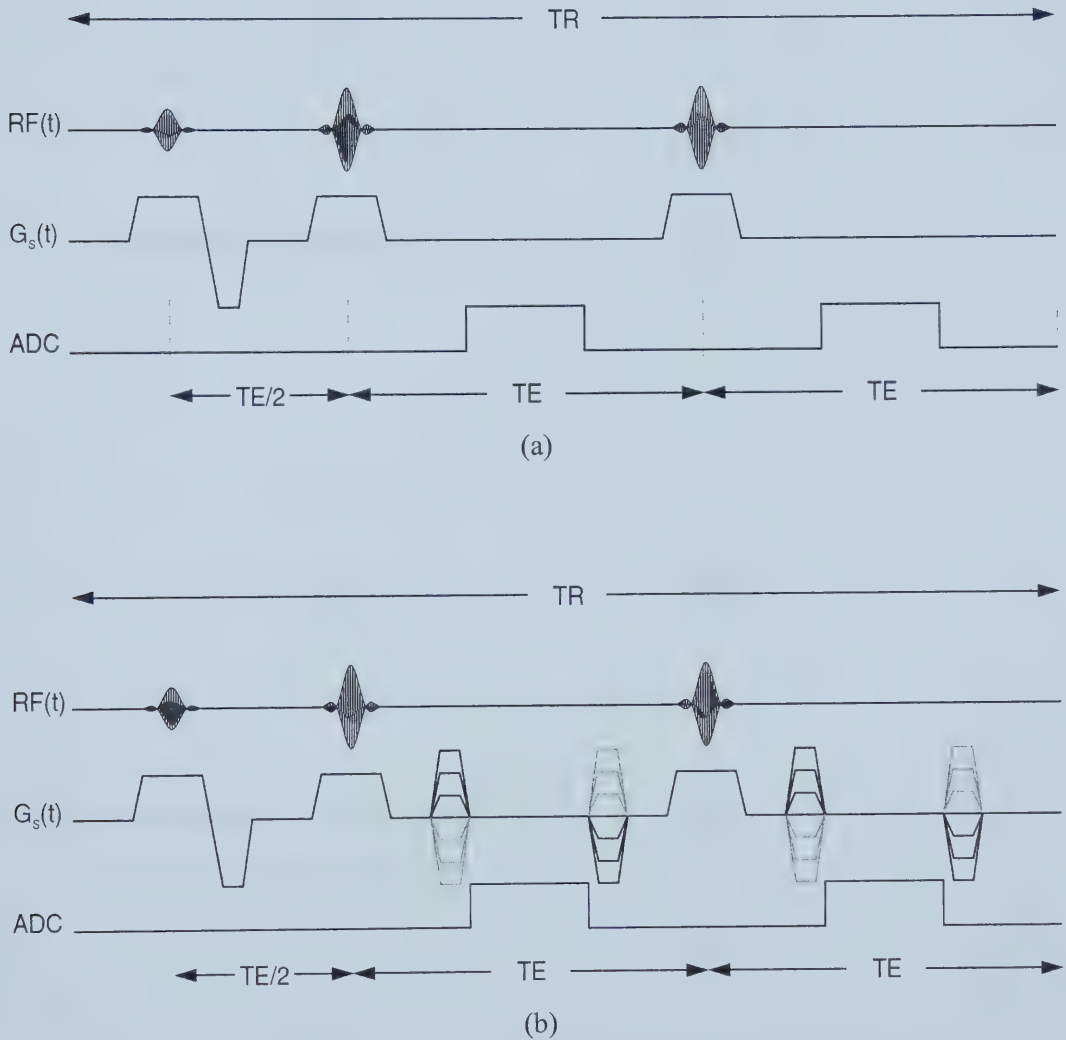


Figure 3.15: Basic two (a) and three (b) dimensional slice gradient waveforms. Both sets have an initial slice selective excitation pulse followed by a slice selective refocus pulse. The 3D sequence has a variable slice phase encode gradient applied before the data is acquired. The slice phase encode is unwrapped before proceeding to the next RF refocus pulse.

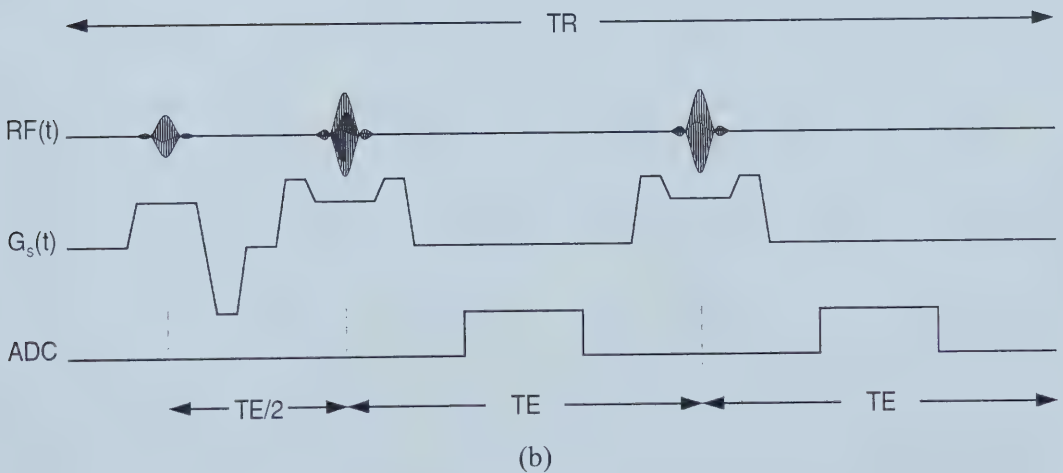
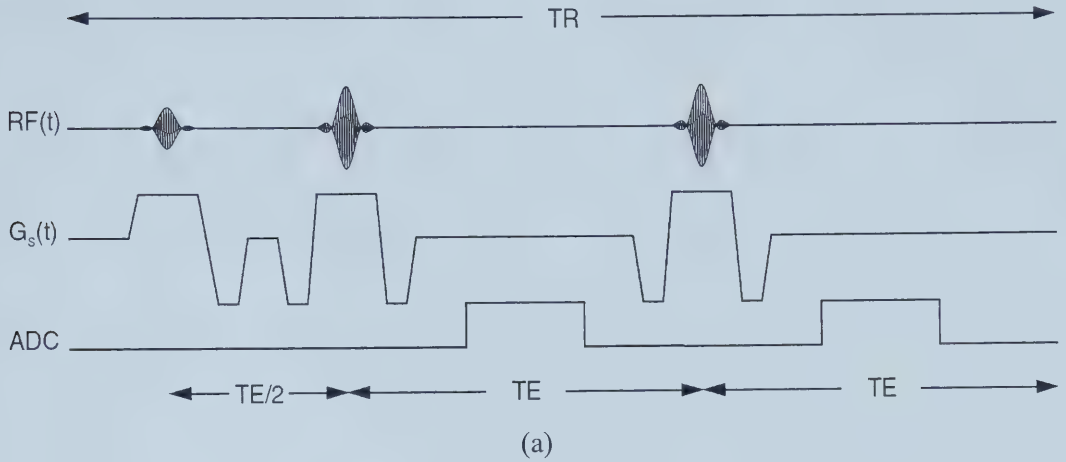


Figure 3.16: Two variations of the 2D slice gradient waveforms. In (a) the slice gradients are flow compensated (nulled 0th and 1st gradient moments) during data acquisition and at the center of RF refocus pulses. This sequence is useful for preserving as much of the transverse magnetization as possible. In (b) the slice gradients only have a nulled 0th moment during data acquisition and have no gradient moment nulling at the center of the RF refocus pulses. This sequence is useful for minimizing the signal intensity from moving spins and stimulated echoes.

3.2.3 Read Gradient

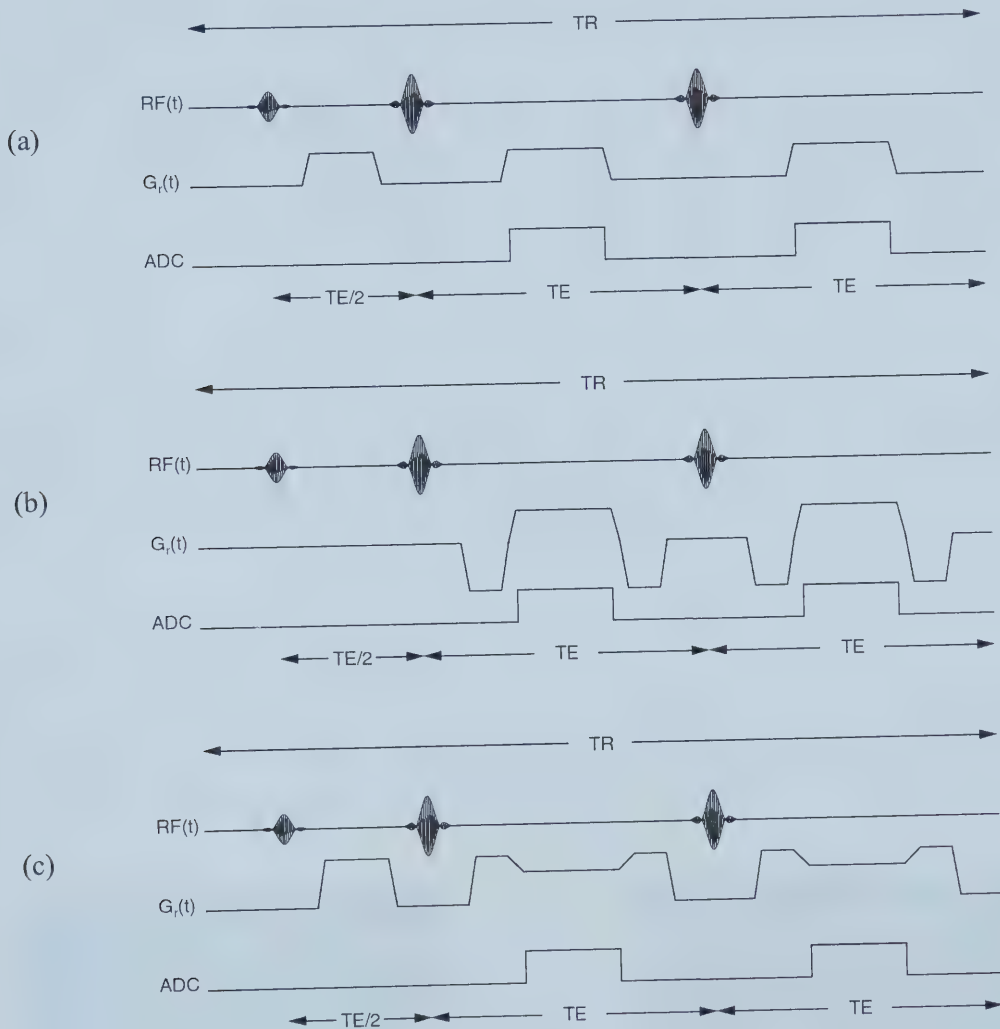


Figure 3.17: Three different read gradient waveforms. The basic sequence in (a) provides no 0^{th} or 1^{st} gradient moment nulling during the RF refocus pulses and only provides a nulled 0^{th} gradient moment at the center of data acquisition. This variation is good for general imaging where moving spins are not of concern and maximum signal is not required. In (b) the read gradient is flow compensated providing 0^{th} and 1^{st} gradient moment nulling during the RF refocus pulses and at the center of data acquisition. This alteration allows maximum signal intensities by preserving stimulated echoes and the contribution from moving spins. The sequence in (c) is the same as the first basic sequence with the exception of extra gradient dephasing. This provides extra suppression of any signal deriving from stimulated echoes or moving spins.

3.2.4 Phase Gradient

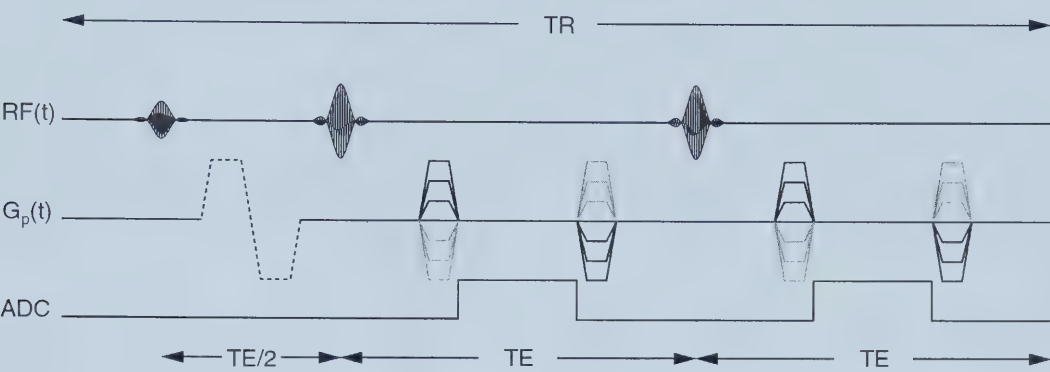


Figure 3.18: The phase encode gradient set for a FSE sequence. The waveforms only provide 0th gradient moment nulling during the RF pulses as well. The optional dashed bipolar gradient between the excitation and refocus pulses can be used to further dephase the signal resulting from stimulated echoes or moving spins.

3.2.5 Phase Encode Order

In the first chapter, the signal equation was used to show that during a MR experiment data is collected in the spatial-spectral domain called k-space.

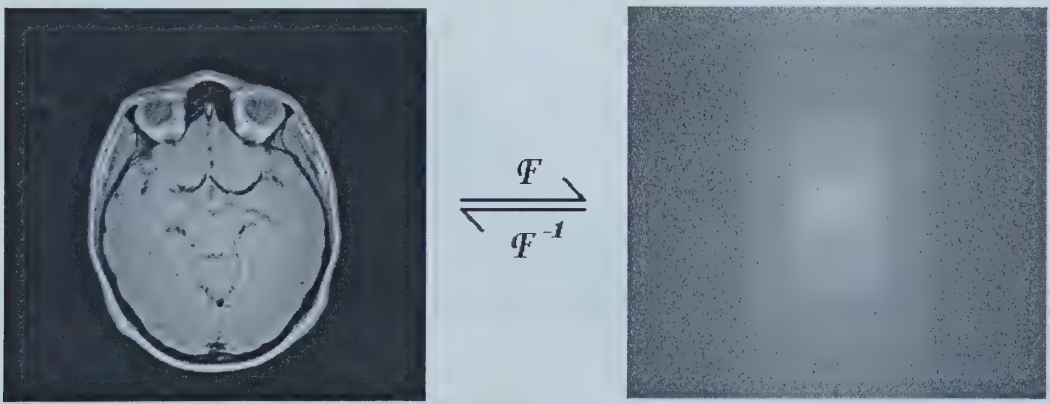


Figure 3.19: The k-space data obtained with a FT imaging sequence is related to the MR image through the Fourier transform.

The trajectory along which k-space data is acquired is referred to as the phase encode order. There are numerous trajectories through k-space that may be used to acquire an MR image (48-56). The Cartesian method of k-space coverage has been selected for this FSE sequence. This method acquires lines or views across k-space with each view corresponding to a particular phase encode value. Since several views are obtained in a single segment, a decision must be made as to the order data will be encoded into k-space.

The central views of k-space will contribute primarily to the contrast of the MR image while the outer views will primarily contribute to the resolution and detail of the image. Thus, encoding certain views to be the center of k-space is one of the several methods of obtaining contrast in FSE sequences (57, 58). The echo that is encoded determines the echo time TE in a FSE sequence. For example, if the echo train spacing is 10 ms and the fourth echo is encoded to be the center of k-space, then $TE = 40$ ms.

To illustrate the phase encode order used in the FSE sequence, consider a FSE sequence with an echo train length of 8. Suppose the 8 echoes have amplitudes represented by the bar graph in Fig. 3.20.

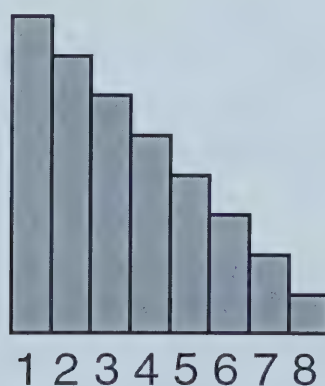


Figure 3.20: The amplitudes of the 8 echoes in a single FSE segment.

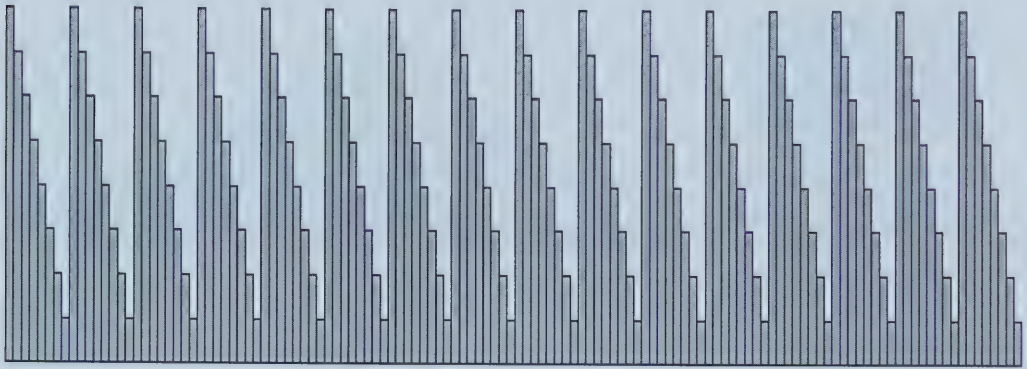


Figure 3.21: An image with 128 phase encode views requires 16 segments of 8 echoes to be acquired. The modulation of echo amplitudes as they appear in time is shown above.

If the phase encode order was acquired linearly across k-space then the data would have the k-space modulation shown in Figure 3.21. This type of k-space modulation would result in an image with severe ghosting. To overcome this problem, the views are phase encoded in an order such that echoes with similar amplitudes are encoded next to each other in k-space. This results in a smoother modulation of the k-space data and a significant reduction in the amount of image ghosting.

Figure 3.22a-c demonstrates the k-space modulation with the first, fourth and eighth echo of the echo train being used as the contrast echo. FSE neck images acquired with the k-space modulations shown in Fig. 3.22a-c are shown in Fig. 3.22d-f. The sagittal neck images are 3mm thick, have a resolution of 256x128 and were acquired with an echo train length of 8 echoes.

In addition to grouping similar echoes, the contrast (TE) can be selected by the appropriate choice of the central echoes. The echo that is encoded at the center of k-space is referred to as the contrast echo. Figure 3.23 shows a series of T₂ weighted head images acquired with FSE at 3.0 T.

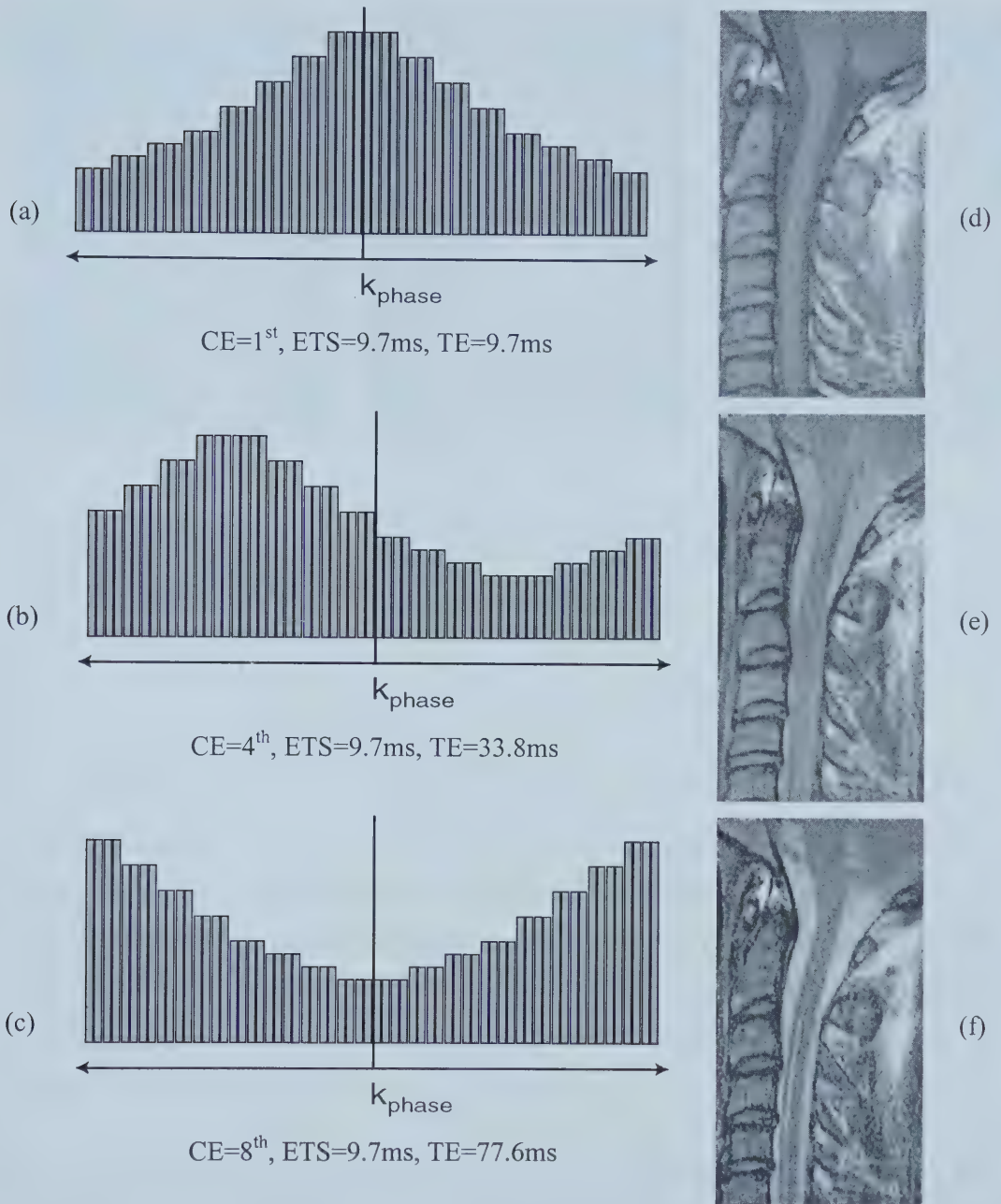


Figure 3.22: The k-space modulations resulting from three different echo times are shown in (a-c). CE is the contrast echo, ETS is the echo train spacing and TE is the resulting echo time. The resulting neck images using the modulations of (a-c) are shown in (d-f).

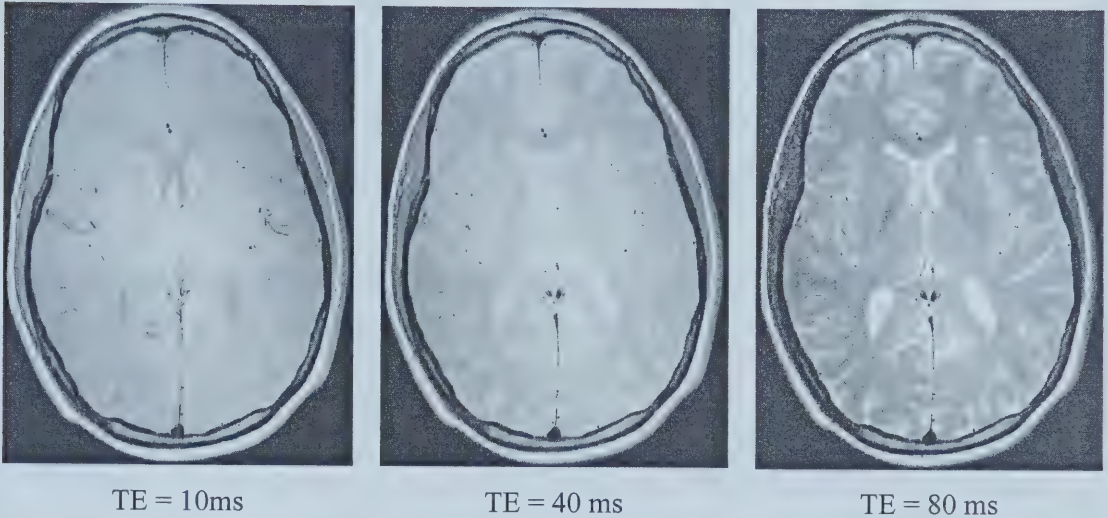


Figure 3.23: Transverse FSE head images acquired at 3.0 T. The images are 512x512 in a 240 mm FOV and a 3 mm slice. The segments had 16 echoes with an ETS of 10 ms and a repetition time of 2000 ms.

3.3 Implementing FSE Sequences

Sections 3.1 and 3.2 have laid out the building blocks for a fast spin echo sequence. The actual combination of RF and gradient properties will depend on the specific MR experiment. Fortunately all of the variations discussed in this chapter can coexist within a single MR sequence and simply be turned on or off depending on the demands of the current application. This makes the FSE sequence extremely robust and diverse in Magnetic Resonance Imaging.

However, as research and clinical scanners move to higher field strengths (> 1.5 T), the use of fast spin echo imaging can require compromises not considered at lower magnetic fields. The next chapter is a theoretical and experimental investigation of one of these problems encountered with FSE sequences at high magnetic field strengths.

3.4 Bibliography

1. Busse RF, Riederer SJ, Fletcher JG, Bharucha AE and Brandt KR. Interactive Fast Spin Echo Imaging. *Magn Reson Med* 2000; 44: 339-348.
2. Hennig J, Nauerth A and Friedburg H. RARE Imaging: A Fast Imaging Method for Clinical MR. *Magn Reson Med* 1986; 3: 823-833.
3. Melki P, Mulkern RV, Dacher JN, Helenon O, Higuchi N, Oshio K, Einstein S, Jolesz F and Pourcelot L. Fast Spin-Echo MRI Techniques - Contrast Characteristics and Clinical Potential. *J Radiol* 1993; 74: 179-190.
4. Plewes DB and Bishop J. Spin-Echo MR Imaging. In: M. Bronskill and P. Sprawls. *The Physics of MRI - 1992 AAPM Summer School Proceedings*. Woodbury: American Institute of Physics, Inc.; 1993. 166-187
5. Bottomley PA and Andrew ER. RF magnetic Field Penetration, Phase Shift and Power Dissipation in Biological Tissue: Implications for NMR Imaging. *Phys Biol Med* 1978; 23: 630.
6. Schaefer DJ. Bioeffects of MRI and Patient Safety. In: M. Bronskill and P. Sprawls. *The Physics of MRI - 1992 AAPM Summer School Proceedings*. Woodbury: American Institute of Physics, Inc.; 1993. 607-646
7. Guidance for Magnetic Resonance Diagnostic Devices. US Food and Drug Administration: Center for Devices and Radiological Health. Available from: <http://www.fda.gov/cdrh/ode/magdev.html>. Accessed February 2003.

8. Alecci M, Collins CM, Smith MB and Jezzard P. Radio Frequency Magnetic Field Mapping of a 3 Tesla Birdcage Coil: Experimental and Theoretical Dependence on Sample Properties. *Magn Reson Med* 2001; 46: 379-385.
9. Hornak JP, Szumowski J and Bryant RG. Magnetic Field Mapping. *Magn Reson Med* 1988; 6: 158-163.
10. Insko EK and Bolinger L. Mapping of the Radio Frequency Field. *J Magn Reson* 1993; 103: 82-85.
11. Kelter JR, Carlson JW, Roos MS, Wong STS, Wong TL and Budinger TF. Electromagnetic Fields of Surface Coil In Vivo NMR at High Frequencies. *Magn Reson Med* 1991; 22: 467-480.
12. Murphy-Boesch J, So GJ and James TL. Precision Mapping of the B1 Field Using the Rotating Frame Experiment. *J Magn Reson* 1987; 73: 293-303.
13. Sled JS and Bruce G. Standing Wave and RF Penetration Artifacts Caused by Elliptical Geometry: An Electrodynamics Analysis of MRI. *IEEE Trans Med Imaging* 1998; 17: 653-662.
14. Topp S, Adalsteinsson E and Spielman DM. Fast Multislice B1 Mapping. *Proc ISMRM* 1997; 281.
15. Yang QX, Wang J, Zhang X, Collins CM, Smith MB, Liu H, Zhu X, Vaughan JT, Ugurbil K and Chen W. Analysis of Wave Behavior in Lossy Dielectric Samples at High Field. *Magn Reson Med* 2002; 47: 982-989.

16. Fakri-Bouchet L, Lapray C and Briguët A. Measurements of the Radio Frequency Field in Magnetic Resonance Coils. *Meas Sci Technol* 1998; 9: 1641-1646.
17. Simmons A, Tofts PS, Barker GJ and Arridge SR. Sources of Intensity Non-uniformity in Spin Echo Images at 1.5 T. *Magn Reson Med* 1994; 32: 121-128.
18. Barker GJ, Simmons, Arridge SR and Tofts PS. A Simple Method for Investigating the Effects of Non-uniformity of Radiofrequency Transmission and Radiofrequency Reception in MRI. *Br J Radiol* 1998; 71: 59-67.
19. Hayes CE, Edelstein WA, Schenck JF, Mueller OM and Eash M. An Efficient, Highly Homogeneous Radiofrequency Coil for Whole Body NMR Imaging. *J Magn Reson* 1985; 63: 622-628.
20. Vaughan JT, Garwood M, Collins CM, Liu W, DelaBarre L, Adriany G, Andersen P, Merkle H, Goebel R, Smith MB and Ugurbil K. 7T vs. 4T: RF Power, Homogeneity, and Signal-to-Noise Comparison in Head Images. *Magn Reson Med* 2001; 46: 24-30.
21. Condon BR, Patterson J, Wyper D, Jenkins A and Hadley DM. Image Non-uniformity in Magnetic Resonance Imaging: Its Magnitude and Methods for Correction. *Br J Radiol* 1987; 60: 83-87.
22. Glover GH, Hayes CE, Pelc NJ, Edelstein WA, Mueller OM, Hart HR, Hardy CJ, O'Donnell M and Barber WD. Comparison of Linear and Circular Polarization for Magnetic Resonance Imaging. *J Magn Reson* 1985; 64: 255-270.

23. Jerschow A and Bodenhausen G. Mapping the B1 Field Distribution with Non Ideal Gradients in a High-Resolution NMR Spectrometer. *J Magn Reson* 1999; 137: 108-115.
24. Collins CM and Smith MB. Signal-to-Noise Ratio and Absorbed Power as Functions of Main Magnetic Field Strength, and definition of "90°" RF Pulse for the Head in the Birdcage Coil. *Magn Reson Med* 2001; 45: 684-691.
25. Hoult DI, Chen C-N and Sank VJ. The Field Dependence of NMR Imaging. *Magn Reson Med* 1986; 3: 730-746.
26. Hoult DI and Phil D. Sensitivity and Power Deposition in a High-Field Imaging Experiment. *J Magn Reson Imaging* 2000; 45: 46-67.
27. Freeman R. Shaped Radiofrequency Pulses in High Resolution NMR. *J NMR Spect* 1998; 32: 59-106.
28. Ro YM and Cho ZH. A Novel Flow Suppression Technique Using Tailored RF Pulses. *Magn Reson Med* 1993; 29: 660-666.
29. Slotboom J, Gunning JW and Mehlkopf AF. Design of Frequency Selective RF Pulses by Optimizing a Small Number of Pulse Parameters. *J Magn Reson Ser A* 1993; 101: 257-264.
30. Xu ZH and Chan AK. A Near Resonance Solution to the Bloch Equations and Its Applications to RF Pulse Design. *J Magn Reson* 1999; 32: 225-231.
31. LeRoux P and Hinks RS. Stabilization of Echo Amplitudes in FSE Sequences. *Magn Reson Med* 1993; 30: 183-190.

32. Mulkern RV, Wong STS, Winalski C and Jolesz FA. Contrast Manipulation and Artifact Assessment of 2D and 3D RARE Sequences. *Magn Reson Imaging* 1990; 8: 557-566.
33. Hennig J. Multiecho Imaging Sequences with Low Refocusing Flip Angles. *J Magn Res* 1988; 78: 397-407.
34. Alsop DC. The Sensitivity of Low Flip Angle RARE Imaging. *Magn Reson Med* 1997; 37: 176-184.
35. Meiboom S and Gill D. Modified Spin-Echo Method for Measuring Nuclear Relaxation Times. *Rev Sci Instr* 1958; 29: 688.
36. Roden MS. *Analog and Digital Communication Systems*. Upper Saddle River: Prentice Hall Inc.; 1996. 560 pages.
37. Oppenheim AV and Schafer RW. *Discrete Time Signal Processing*. Englewood Cliffs: Prentice Hall Inc.; 1989.
38. Mendes JK, DeZanche N, Emery DJ and Wilman AH. Whole Body Fast Spin Echo Imaging at Three Tesla. *Proc ISMRM* 2001; 2014.
39. Abragam A. *The Principles of Nuclear Magnetism: The International Series of Monographs on Physics*. Oxford: Clarendon Press; 1961. 599 pages.
40. Block W, Pauly J, Kerr A and Nishimura D. Consistent Fat Suppression with Compensated Spectral Spatial Pulses. *Magn Reson Med* 1997; 38: 198-206.

41. Delfaut EM, Beltran J, Johnson G, Rousseau J, Marchandise X and Cotten A. Fat Suppression in MR Imaging: Techniques and Pitfalls. *Sci Exhibit* 1999; 19: 373-382.
42. Higuchi N, Hiramatsu K and Mulkern RV. A Novel Method for Fat Suppression in RARE Sequences. *Magn Reson Med* 1992; 27: 107-117.
43. Thomas SD, Al-Kwif O, Emery DJ and Wilman AH. Application of Magnetization Transfer at 3.0 T in Three-Dimensional Time-of-Flight Magnetic Resonance Angiography of the Intracranial Arteries. *J Magn Reson Imaging* 2002; 15: 479-483.
44. Thomas SR. Magnets and Gradient Coils: Types and Characteristics. In: M. Bronskill and P. Sprawls. *The Physics of MRI - 1992 AAPM Summer School Proceedings*. Woodbury: American Institute of Physics, Inc.; 1993. 56-97
45. Irnich W and Schmitt F. Magnetostimulation in MRI. *Magn Reson Med* 1995; 33: 619-623.
46. Zhao HW, Crozier S and Liu F. Finite Difference Time Domain (FDTD) Method for Modeling the Effect of Switched Gradients on the Human Body in MRI. *Magn Reson Med* 2002; 48: 1037-1042.
47. Hoffmann A, Faber SC, Werhahn KJ, Jager L and Reiser M. Electromyography in MRI - First Recordings of Peripheral Nerve Activation Caused by Fast Magnetic Field Gradients. *Magn Reson Med* 2000; 43: 534-539.

48. Wilman AH and Riederer SJ. Improved Centric Phase Encoding Orders for Three-Dimensional Magnetization-Prepared MR Angiography. *Magn Reson Med* 1996; 36: 384-392.
49. Wilman AH and Riederer SJ. Performance of an Elliptical Centric View Order for Signal Enhancement and Motion Artifact Suppression in Breath-hold Three-Dimensional Gradient Echo Imaging. *Magn Reson Med* 1997; 38: 793-802.
50. Glover GH. Simple Analytic Spiral K-Space Algorithm. *Magn Reson Med* 1999; 42: 412-415.
51. Kholmovski EG, Parker DL and Alexander AL. A Generalized K-Sampling Scheme for 3D Fast Spin Echo. *J Magn Reson Imaging* 2000; 11: 549-558.
52. McGibney, Smith MR, Nichols ST and Crawley A. Quantitative Evaluation of Several Partial Fourier Reconstruction Algorithms Used in MRI. *Magn Reson Med* 1993; 30: 51-59.
53. Pipe JG. An Optimized Center-Out K-Space Trajectory for Multishot MRI: Comparison With Spiral and Projection Reconstruction. *Magn Reson Med* 1999; 42: 714-720.
54. Hinks RS and Constable RT. Gradient Moment Nulling in Fast Spin Echo. *Magn Reson Med* 1994; 32: 698-706.
55. Bornert P, Stuber M, Botnar RM, Kissinger KV, Koken P, Spuentrup E and Manning WJ. Direct Comparison of 3D Spiral Vs. Cartesian Gradient-Echo Coronary Magnetic Resonance Angiography. *Magn Reson Med* 2001; 46: 789-794.

56. Keller PJ, Heiserman JE, Fram EK, Rand SD and Drayer BP. A Nyquist Modulated Echo-to-View Mapping Scheme for Fast Spin Echo Imaging. *Magn Reson Med* 1995; 33: 838-842.
57. Fautz HP, Buchert M, Husstedt H, Laubenberger J and Hennig J. TSE Sequences with Spin Echo Contrast. *Magn Reson Med* 2000; 43: 577-582.
58. Constable RT, Anderson AW, Zhong J and Gore JC. Factors Influencing Contrast in Fast Spin Echo MR Imaging. *Magn Reson Imaging* 1992; 10: 497-511.

4

Fast Spin Echo Angiography

Magnetic Resonance Angiography (MRA) is the application of Magnetic Resonance Imaging to the visualization of blood vessels. MRA images are often classified as either “white” or “black” blood images. White blood techniques typically saturate the background tissue resulting in greater signal intensities from spins in the blood than spins in the surrounding tissue. Black blood imaging differs by minimizing the signal from the spins in blood while maximizing the surrounding tissue signal.

It is possible, but not very common, to utilize a FSE sequence in certain applications of white blood imaging (1-4). However, the FSE sequence is more suitable and more established as a black blood imaging technique (5-13). The black blood technique is particularly useful for vessel wall imaging as it preserves the signal from the surrounding tissue (14-17). This chapter examines some of the principles associated with the implementation of FSE black blood imaging.

4.1 Black Blood Imaging

4.1.1 Definition of Black Blood

The color label of a MRA technique refers to the signal intensity of the blood in the image. MR images are usually displayed with a gray scale that associates white with the highest signal intensity and black with the lowest signal intensity. White blood images attempt to create blood vessels that have maximum signal intensity with surrounding tissue saturated to a minimum value. Black blood images attempt to reduce the blood signal as much as possible while maintaining a maximum signal from the background (Fig. 4.1).

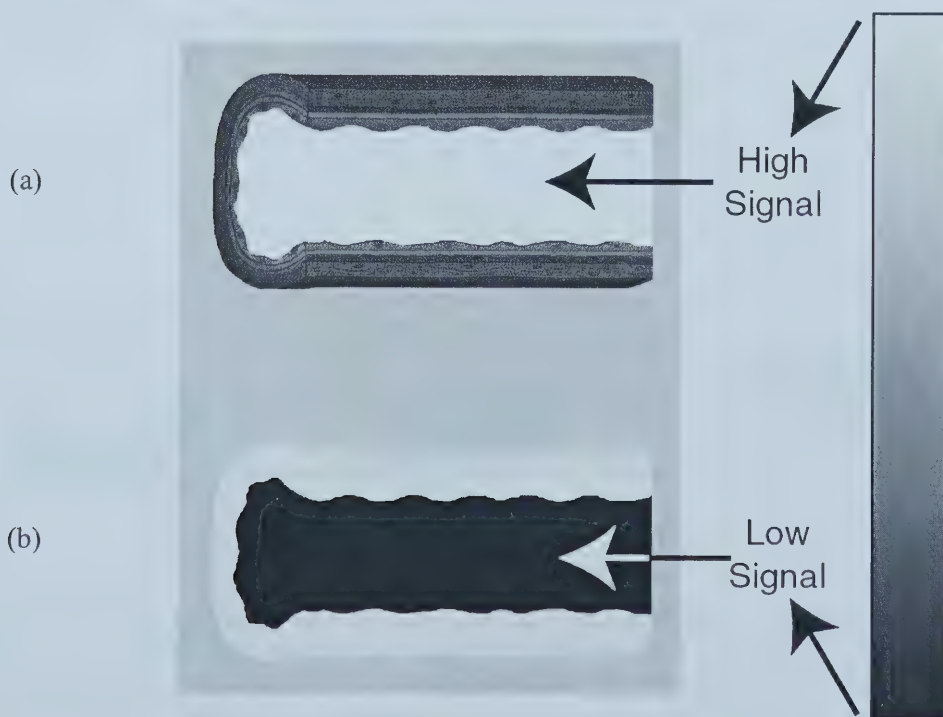


Figure 4.1: In a white blood image (a), the high intensity blood signal is contrasted with the reduced signal of the saturated background tissue. The black blood image in (b) has bright surrounding tissue while the signal from blood is reduced to a minimum.

4.1.2 Signal Saturation

Equation [1.25] demonstrated that once a magnetization vector was excited, there was a necessary time period governed by T_1 for the magnetization to return to thermal equilibrium. If the magnetization vector is excited before it returns to thermal equilibrium then the resulting transverse magnetization will only be a fraction of the thermal equilibrium value. A spin is saturated if little or no signal results from an excitation because the spin is still recovering from a previous excitation.

In Angiography, saturation involves exciting a slice of spins offset from the imaging slice in a direction opposite the blood flow. There is then a time delay to allow the excited blood spins to flow into the imaging slice at which time the image slice is excited. The spins from the blood have not fully recovered from the previous saturation pulse and will consequently have a lowered transverse magnetization (Fig. 4.2).

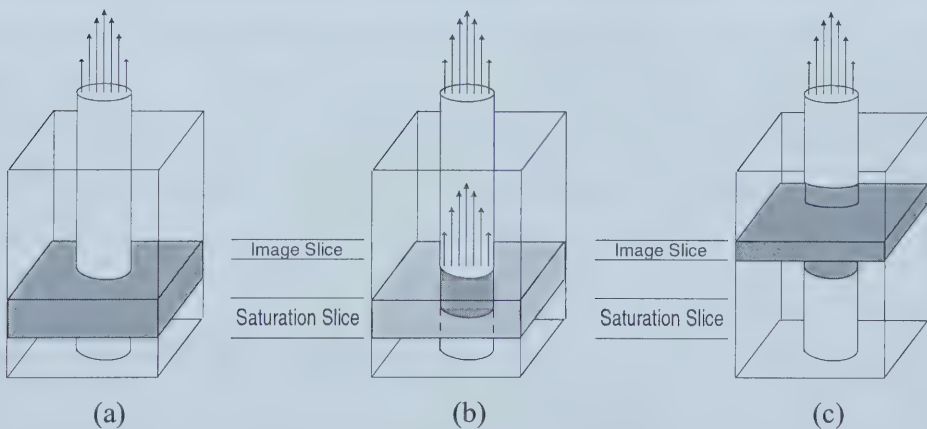


Figure 4.2: In (a) a slice of spins are fully excited by a saturation pulse. The saturated spins are then allowed to flow into the image slice (b). After the short delay, the slice is imaged as normal. The spins in the stationary tissue did not receive the saturation RF pulse and will have full signal intensity. The spins from the blood that received the saturation pulse will have not fully recovered and will suffer signal degradation.

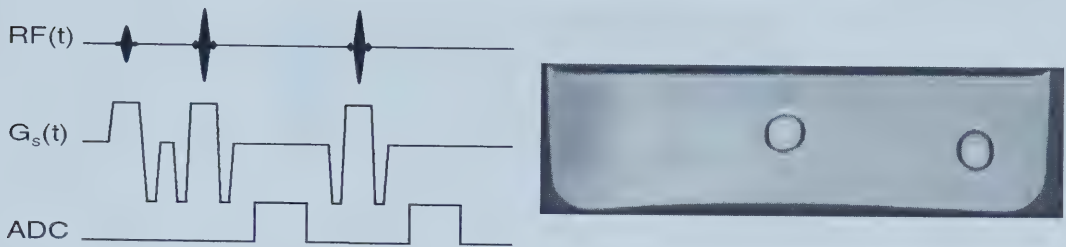
White blood imaging uses a similar technique except the saturation slab and the image slab are not offset from one another. As a result, the background tissue is saturated but any fresh blood that flows into the image slice during the delay will be fully excited. This white blood technique is difficult with FSE sequences since it incompatibly requires blood to travel quickly for the saturated spins to exit the slice, but slowly so that a fresh spin within the slice for the excitation pulse remains there for each refocus pulse. This is one of the reasons that FSE is better suited to black blood imaging.

One problem with relying on saturation for contrast is that technique is dependent on the velocity of the blood. In white blood imaging, a slow blood flow means not enough fresh blood enters the slice and the blood signal intensity is also saturated. For black blood imaging, only the spins within a specified range of velocities will appear inside the image slice during excitation. Spins with slower flow will not make it to the image slice while spins with greater velocities will pass completely through the image slice.

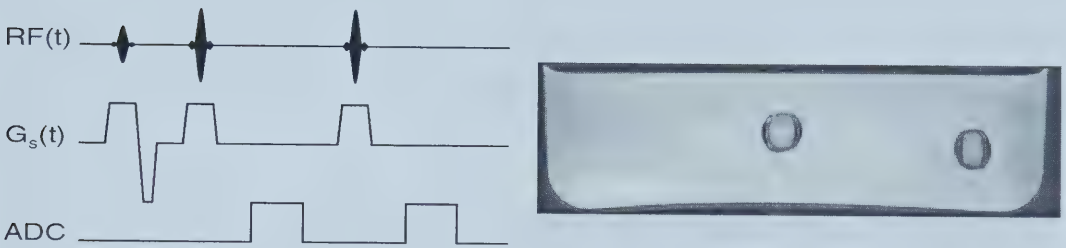
The advantage of black blood imaging is that additional methods of gaining contrast can be applied later in the sequence to more fully reduce the signal intensity of the blood and improve contrast. A black blood imaging sequence should therefore seek to further dephase the spins in blood but refrain from dephasing the spins in stationary tissue. This is accomplished through careful planning of gradient moment nulling.

4.1.3 Gradient Configurations

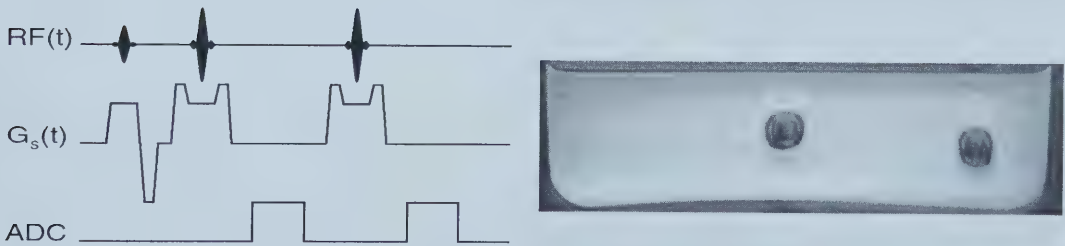
To preserve the signal of the surrounding tissue, all gradients should have a nulled 0th gradient moments but the 1st gradient moments should be maximized so any moving spins will acquire a phase distribution. Figure 4.3 investigates the effect of gradient moment nulling in the slice direction.



(a)



(b)



(c)

Figure 4.3: Various slice FSE gradient configurations and the effects on flowing spins are demonstrated. The flow rate was 1.7 L/min and the images have an echo time of 10 ms with a 8-echo segment. The pixel size is 1x1 mm with a 3mm slice thickness and a repetition time of 2000 ms. The slice gradient in (a) has both 0th and 1st gradient moment nulling and consequently is a poor choice for black blood imaging. The standard FSE slice gradient in (b) show improved blood dephasing. The best choice is the gradient in (c) that offers a nulled 0th gradient moment but maximizes the 1st gradient moment. Expanded views of the gradients are found in Fig. 3.15 and 3.16.

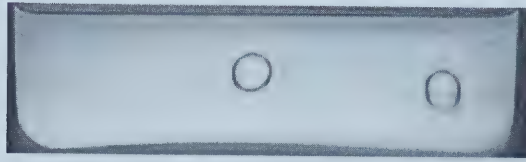
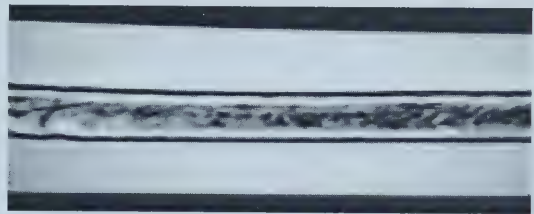
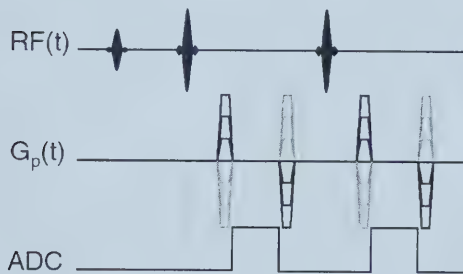
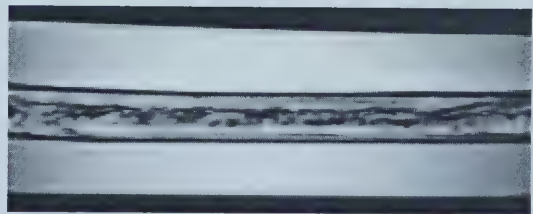
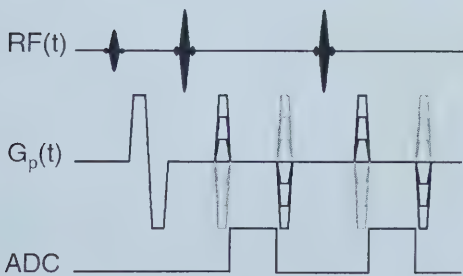


Figure 4.4: Ghosting in the image around the tube is partially due to the susceptibility of the plastic tube itself. This image is acquired with the same sequence and parameters as Fig. 4.3c except the flow has been stopped.

The ghosting in Fig. 4.3 around the flowing spins is partially due to the flow itself and partially due to the susceptibility of the plastic in the tube itself. A similar comparison can be made for the phase and read gradients.

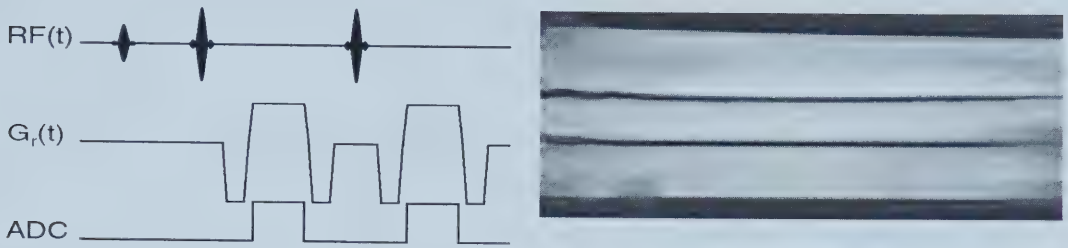


(a)

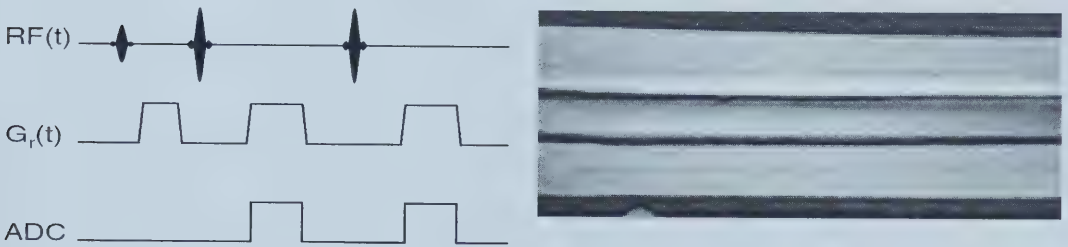


(b)

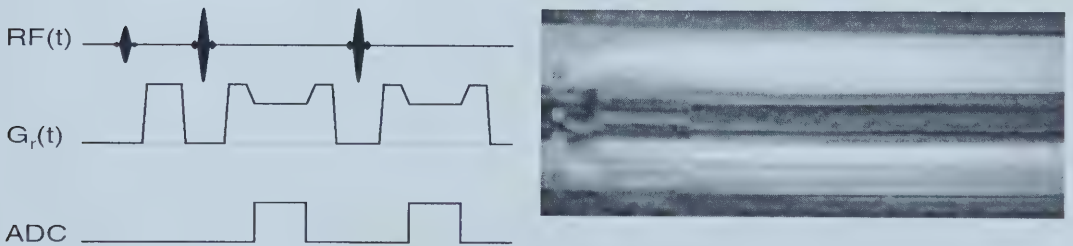
Figure 4.5: Various phase FSE gradient configurations and the effects on flowing spins are demonstrated. The flow rate was 1.7 L/min and the images have an echo time of 10 ms with a 8-echo segment. The pixel size is 1x1 mm with a 3mm slice thickness and a repetition time of 2000 ms. The phase configuration with the additional bipolar gradient (b) shows improved flow suppression over the configuration in (a). An expanded view of the gradient is found in Fig. 3.18.



(a)



(b)



(c)

Figure 4.6: Various read FSE gradient configurations and the effects on flowing spins are demonstrated. The flow rate was 1.7 L/min and the images have an echo time of 10 ms with a 8-echo segment. The pixel size is 1x1 mm with a 3mm slice thickness and a repetition time of 2000 ms. The flow compensated read gradient in (a) is obviously a poor choice for black blood imaging. The standard FSE slice gradient in (b) shows marginal improvement over the flow compensated configuration. The extra gradients in (c) provide the needed 1st gradient moment and allow good blood signal suppression. Expanded views of the gradients are found in Fig. 3.17.

The final gradient configuration that will optimize the signal reduction in black blood imaging is shown in Fig. 4.7. Although only 1st gradient moments have been carefully examined, it can also be shown that this gradient set will also provide higher degrees of gradient moment dephasing. It has already been shown that under conditions of slow or no flow, traditional white blood techniques suffer from saturation effects and have a reduction in blood to background contrast. A second difficulty with white blood imaging is that the related sequences are generally only 0th and 1st gradient moment nulled. This is an important fact since the velocity of blood varies spatially within a vessel and has a time dependence associated with the heartbeat. For complex or random blood flows, such as those found near a stenosis of a diseased vessel, the lack of higher moment nulling will result in a loss of contrast in white blood imaging. In contrast, with black blood imaging the amount of contrast increases with the turbulent flow since higher moments are also dephased by the FSE sequence.

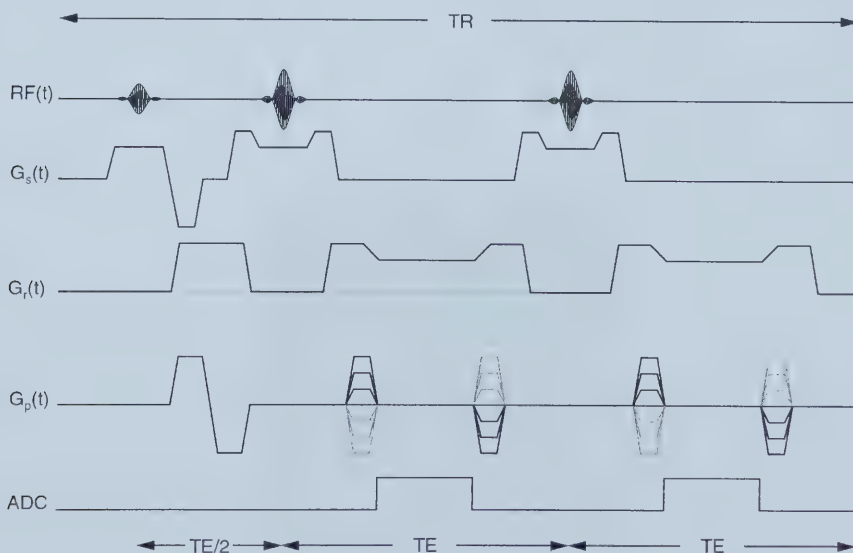


Figure 4.7: A set gradients ideal for black blood imaging. The slice gradient contains extra lobes on either side of the refocus pulse to increase the 1st gradient moment. The read gradient contains extra lobes on either side of the data acquire window to increase the 1st gradient moment. The phase encode gradient contains a bipolar pulse at the start of the sequence to increase the 1st gradient moment. All the gradients have nulled 0th gradient moments during at the center of data acquisition.

4.1.4 Black Blood Images

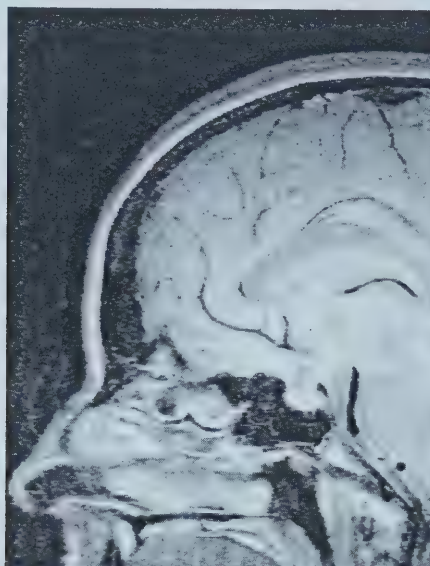


Figure 4.8: 3.0 T cranial FSE black blood image. 256x256, 5 mm slice, 240 mm FOV, 8-echo segment, TE=15 ms and TR=1500 ms.

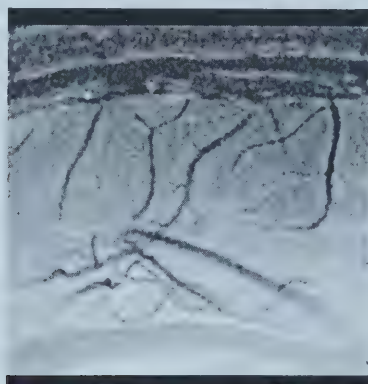


Figure 4.9: Close up view of a 3.0 T cranial black blood image. 256x256, 5 mm slice, 240 mm FOV, 8-echo segment, TE=15 ms and TR=1500 ms.

4.2 Future Optimizations

4.2.1 Echo Time

One of the advantages of using FSE for black blood imaging is that the blood continues to be dephased throughout the entire segment. The total amount of phase distribution from the 1st gradient moments builds with each echo. In addition, the spins that were initially excited inside the slice will eventually flow out of the slice and will no longer experience the refocus pulses. From the point of view of blood dephasing, the longer the echo time the better. However, a longer echo time also results in lower background signal due to relaxation. It is therefore essential to choose an optimal echo time that balances blood and background signal levels.

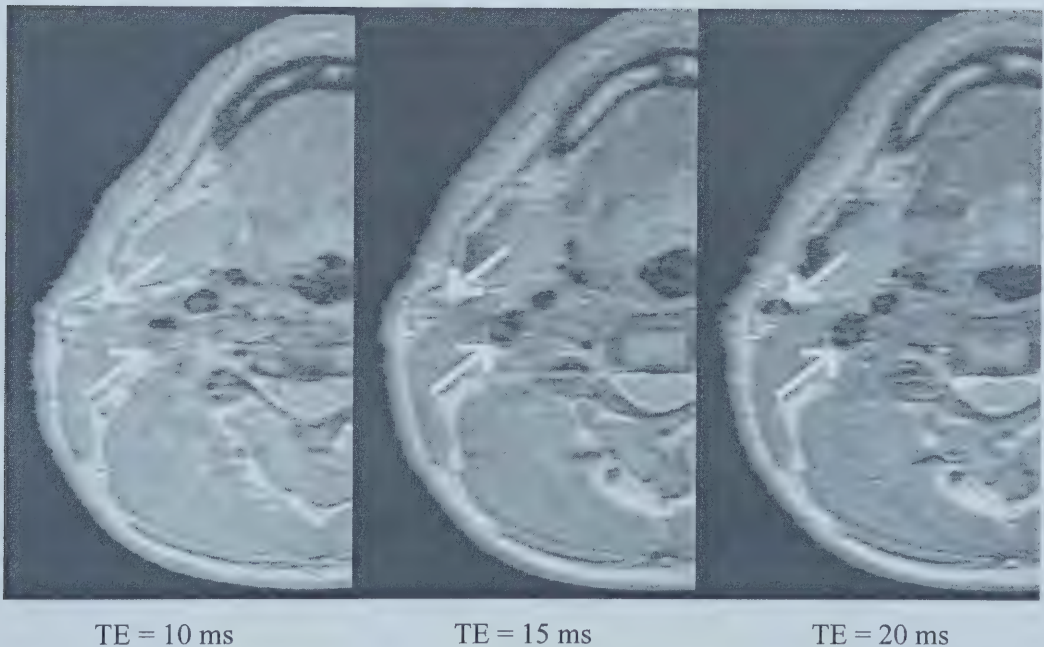


Figure 4.10: The effect of echo time on black blood images is toward improved blood suppression but also increased background signal loss. The 512x256 FSE images were acquired in a 220 mm FOV with a 3mm slice thickness. The echo time was changed via the echo train spacing, with the first echo of the 16-echo segment being used as the contrast echo. In this case, a 20 ms echo time optimizes blood suppression with SNR.

4.2.2 Hyperechoes

In optimizing the echo time, the main focus so far has been on minimizing the signal from the blood. It is also beneficial to consider methods that allow the maximization of the background signal, particularly during the encoding of the contrast echo. These methods usually involve the reduction of refocus flip angles that will lead to an increasingly complex formation of stimulated echoes.

The hyperecho mechanism is a recently introduced method in which a system of magnetization vectors subjected to a spin echo sequence of arbitrarily reduced refocus flip angles can be made to form a spin echo with full intensity (18). The spin echo formed can be regarded as an echo of echoes and is labeled a hyperecho. The application to black blood imaging is the hope that the stationary tissue will experience all of the refocus pulses in a hyperecho train, but the flowing blood will only experience a fraction of the RF pulses. If the stationary tissue forms a hyperecho during the contrast echo and the flowing blood does not, then the contrast will be improved.

To demonstrate the basic formation of a hyperecho, rotation operators about the x-, y- and z-axis must be first described (Fig 4.11). These rotation operators will be used to define the basic symmetry relations that form the foundation of hyperechoes.

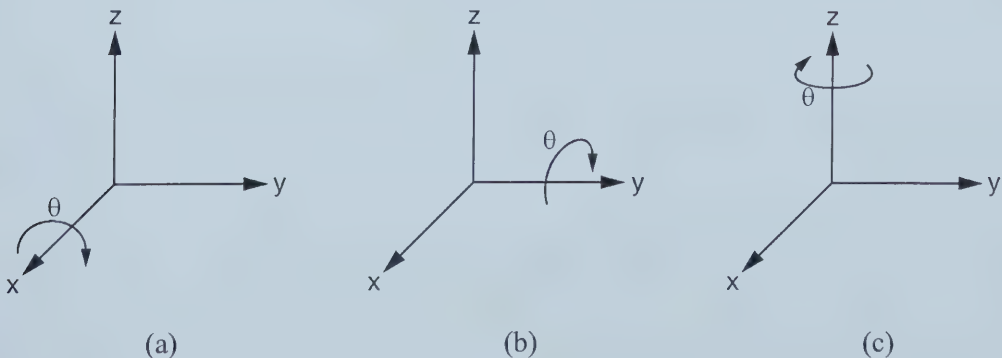


Figure 4.11: Graphical representation of (a) $R_x(\theta)$, (b) $R_y(\theta)$ and (c) $R_z(\theta)$.

The mathematical representations of the rotation operators are:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix} \quad [4.1a]$$

$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix} \quad [4.1b]$$

$$R_z(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [4.1c]$$

Generalizing Eq. [4.1], the rotation about an arbitrary axis in the xy-plane (defined by an angle ϕ) is:

$$R_\phi(\theta) = R_z(\phi)R_y(\theta)R_z(-\phi) \quad [4.2]$$

The hyper echo mechanism is based on a set of basic symmetries (19). The first symmetry is the familiar situation observed in the formation of a spin echo:

$$R_z(\phi)R_y(\pi)R_z(\phi) = R_y(\pi) \quad [4.3]$$

The second symmetry relies on the fact that subsequent rotations about the same axis are additive:

$$R_y(-\theta)R_y(\pi)R_y(\theta) = R_y(\pi) \quad [4.4]$$

The third symmetry is a combination of the first two symmetries and Eq. [4.2] such that:

$$R_{-\phi}(-\theta)R_y(\pi)R_{\phi}(\theta) = R_y(\pi) \quad [4.5]$$

The symmetries in Eqs. [4.3-4.5] can be combined and expanded generally as:

$$R_{-\phi_n}(-\theta_n)R_z(\varphi_n)\dots R_{-\phi_1}(-\theta_1)R_z(\varphi_1)R_y(\pi)R_z(\varphi_1)R_{\phi_1}(\theta_1)\dots R_z(\varphi_n)R_{\phi_n}(\theta_n) = R_y(\pi) \quad [4.6]$$

Equation [4.6] can be extended to the application of RF pulses in a spin echo sequence. The j^{th} RF pulse with a flip angle of θ_j and a phase ϕ_j is:

$$P(\theta_j, \phi_j) = R_{\phi_j}(\theta_j) \quad [4.7]$$

where $P(\theta, \phi)$ represents a RF pulse with a flip angle θ and a phase ϕ . The phase acquired between the j^{th} and $j+1^{\text{th}}$ RF pulse is:

$$\varphi(\Delta\omega_j, \tau_j) \quad [4.8]$$

where $\Delta\omega_j$ is the offset frequency and τ_j is interval between the j^{th} and $j+1^{\text{th}}$ RF pulse. Equation [4.6] applied to a spin echo sequence becomes:

$$P(-\theta_n, -\phi_n)R_z(\varphi_n)\dots R_y(\pi)\dots R_z(\varphi_n)P(\theta_n, \phi_n) = R_y(\pi) \quad [4.9]$$

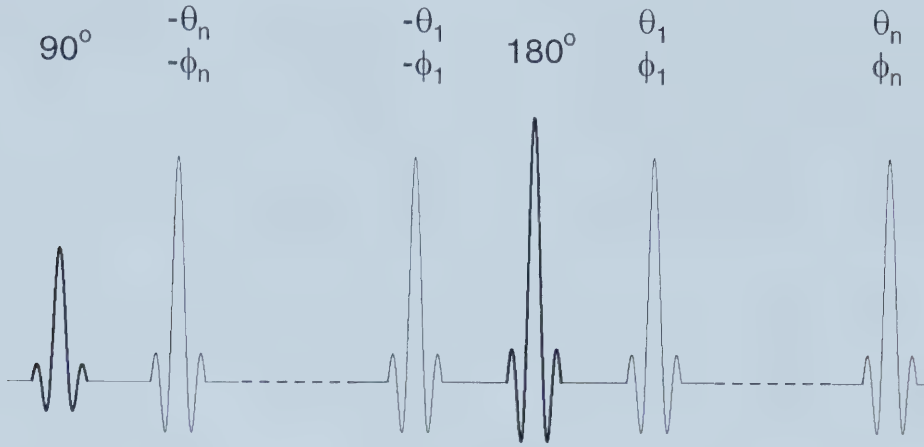


Figure 4.12: A RF pulse diagram of a generalized hyperecho sequence.

If the $N = 2n + 1$ RF pulses are symmetric about the central π -refocus pulse (Fig. 4.12):

$$P = \begin{cases} P(-\theta_{-j}, -\phi_{-j}) & j < 0 \\ P(\pi, 0) & j = 0 \\ P(\theta_j, \phi_j) & j > 0 \end{cases} \quad j = -n, \dots, -1, 0, 1, \dots, n \quad [4.10]$$

and

$$\varphi(\Delta\omega_j, \tau_j) = \varphi(\Delta\omega_{N-j}, \tau_{N-j}) \quad [4.11]$$

then the total effect of the train of RF pulses is equivalent to a single π -refocus pulse. A sequence containing $N = 2n + 1$ arbitrary flip angle refocus pulses can generate a full intensity echo, called a hyperecho, after the N^{th} pulse.

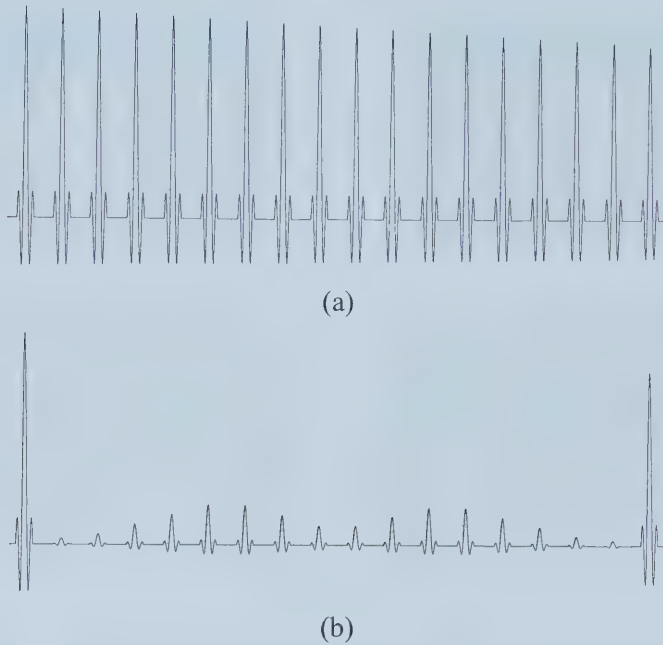


Figure 4.13: The signal intensities expected from a train of (180° flip angle, 0° RF phase) refocus pulses are shown in (a) with the formation of a hyperecho shown in (b). The first nine pulses, before the central π refocus pulse, in the hyperecho sequence are $(180^\circ, 0^\circ)$, $(23^\circ, 80^\circ)$, $(12^\circ, 70^\circ)$, $(77^\circ, 60^\circ)$, $(123^\circ, 50^\circ)$, $(51^\circ, 40^\circ)$, $(34^\circ, 30^\circ)$, $(221^\circ, 20^\circ)$ and $(90^\circ, 10^\circ)$ where (θ, ϕ) represents the (flip angle, RF pulse phase).

4.2.3 Magnetization Transfer

In white blood imaging, magnetization transfer (§3.1.9) is utilized to further saturate the background signal that directly results in an increased contrast (20). In black blood imaging, the use of magnetization transfer may have a benefit in artifact assessment.

For slowly moving blood or in an area with a very uniform blood flow, black blood imaging begins to lose contrast. The blood signal cannot be sufficiently dephased or saturated to differentiate it from the surrounding tissue. In such cases, a second image may need to be acquired so that additional information about the object is obtained.

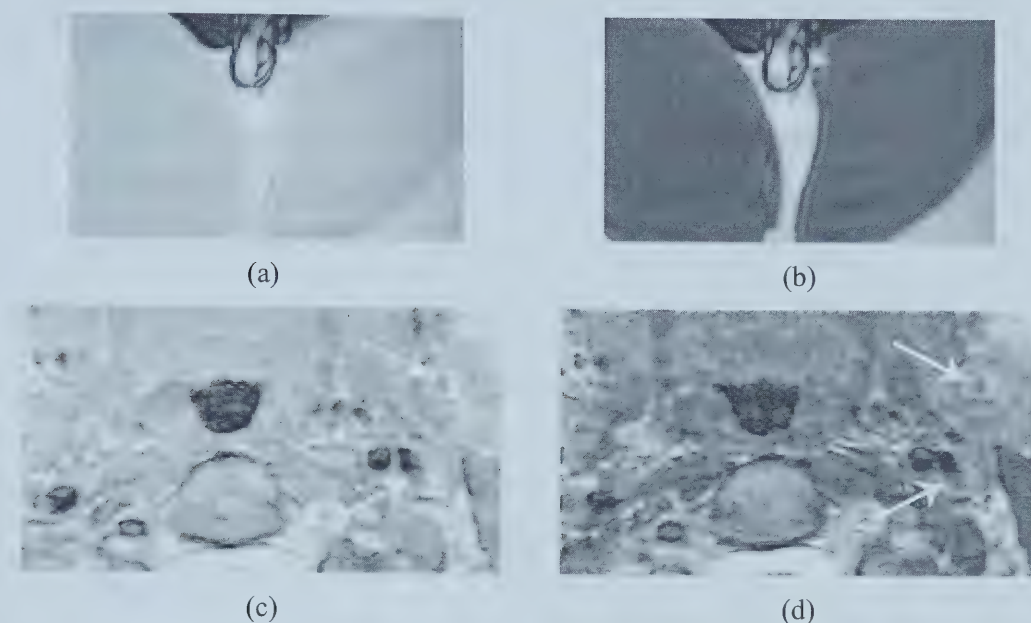


Figure 4.14: The application of a magnetization transfer pulse can be used to identify regions containing macromolecules. In a phantom (a), the blood flow (top middle) is not fully dephased by the black blood FSE sequence because the flow rate is too low. The application of a MT pulse in (b) indicates that the flow artifact does not contain macromolecules, but neither does the water directly underneath the tube. A similar in-vivo example with no MT (c) and with a MT pulse (d) is shown for a section of a transverse slice through a neck. The two flow artifacts identified do not contain any macromolecules.

Since blood does not contain macromolecules, it will not suffer any indirect saturation due to the application of a MT pulse. Muscle, gray matter and white matter are just a few of the tissues that do contain macromolecules and will consequently have reduced signal intensity with the application of a MT pulse. In areas where blood is the only tissue that does not contain macromolecules, a MT pulse can clearly identify the regions where blood has not been adequately dephased enough. This technique would work perfectly if it weren't for the fact that fat does not contain a significant amount of macromolecules and often exists in or near the vessel wall. It is, however, possible to also apply a fat saturation pulse to rid the second image of the fat signal as well. With a combination of MT and fat saturation, it may be possible to accurately image and identify blood vessels using FSE black blood imaging.

4.3 Bibliography

1. Machida Y, Kanazawa H, Kassai Y, Takai H, Sugiura S and Miyazaki M. Effects of Gradient Moment Nulling in 3D Half-Fourier FSE "Bright Blood" Imaging. Proc ISMRM 2000; 1829.
2. Miyazaki M, Ichinose N and Sugiura S. Fresh Blood Contrast 3D-MRA Within Single Breathhold. Proc ISMRM 1998; 780.
3. Kaandorp DW, Wijn PFF and Kopinga K. Three-Dimensional Flow Independent Angiography of Aortic Aneurysms Using Standard Fast Spin Echo. Proc ISMRM 1998; 792.
4. Kassai Y, Miyazaki M, Sugiura S, Makita J and Yamagata H. 3D Half-Fourier RARE with MTC for Cardiac Imaging. Proc ISMRM 1998; 806.
5. Alexander AL, Buswell HR, Sun Y, Chapman BE, Tsuruda JS and Parker DL. Intracranial Black-Blood MR Angiography with High-Resolution 3D Fast Spin Echo. Magn Reson Med 1998; 40: 298-310.
6. Jara H, Yu BC, Caruthers SD, Melhem ER and Yucel EK. Voxel Sensitivity Function Description of Flow-Induced Signal Loss in MR Imaging: Implications for Black-Blood MR Angiography with Turbo Spin Echo Sequences. Magn Reson Med 1999; 41: 575-590.
7. Alexander AL, Chapman BE, Tsuruda JS and Parker DL. A Median Filter for 3D Fast Spin Echo Black Blood Images of Cerebral Vessels. Magn Reson Med 2000; 43: 310-313.

8. Edelman RR, Chien D and Kim D. Fast Selective Black Blood MR Imaging. *Radiol* 1991; 181: 655-660.
9. Liu K and Loncar M. Black Blood Cerebral MRA with a Multi-Contrast SIMVA Acquisition. *Proc ISMRM* 2000; 1498.
10. Liu K and Margosian P. Utilizing Supplementray Flow Information in Dual Contrast SIMVA for Black Blood MRA. *Proc ISMRM* 2000; 1541.
11. Jara H, Kouwenhoven M, Poon E, Bhattia R, Wehrli FW and Yucel EK. A Variable Flip Angle 3D TSE Technique for Black Blood MR Angiography. *Proc ISMRM* 1995; 500.
12. Jara H, Kouwenhoven M, Yu B and Yucel EK. Accurate Measurement of Percent Stenosis with Black Blood 3D Hybrid RARE in the Presence of Turbulent Flow. *Proc ISMRM* 1996; 1253.
13. Ro YM and Cho ZH. A Novel Flow Suppression Technique Using Tailored RF Pulses. *Magn Reson Med* 1993; 29: 660-666.
14. Steinman DA and Rutt BK. On the Nature and Reduction of Plaque-Mimicking Flow Artifacts in Black Blood MRI of the Carotid Bifurcation. *Magn Reson Med* 1998; 39: 635-641.
15. Yuan C, Tsuruda JS, Beach KN, Hayes CE, Ferguson MS, Alpers CE, Foo TK and Strandness DE. Techniques for High Resolution MR Imaging of Atherosclerotic Plaque. *J Magn Reson Imaging* 1994; 4: 43-49.

16. Fayad ZA, Fuster V, Fallon J, Jayasundera T, Worthley SG, Helft G, Aguinaldo G, Badimon JJ and Sharma SK. Noninvasive In Vivo Human Coronary Artery Lumen and Wall Imaging Using Black Blood Magnetic Resonance Imaging. *Circulation* 2000; 102: 506-510.
17. Martin AJ, Gotlieb AI and Henkelman RM. High Resolution MR Imaging of Human Arteries. *J Magn Reson Imaging* 1995; 5: 93-100.
18. Hennig J and Scheffler K. Hyperechoes. *Magn Reson Med* 2001; 46: 6-12.
19. Hennig J and Scheffler K. Hyperechoes - Basic Principles and Applications. *Proceedings of the 9th Annual Meeting of ISMRM* 2001; 437.
20. Thomas SD, Al-Kwafi O, Emery DJ and Wilman AH. Application of Magnetization Transfer at 3.0 T in Three-Dimensional Time-of-Flight Magnetic Resonance Angiography of the Intracranial Arteries. *J Magn Reson Imaging* 2002; 15: 479-483.

5

RF Homogeneity and Reduced Flip Angles[†]

Magnetic Resonance Imaging techniques based on fast spin echo (FSE) are widely used. At magnetic fields greater than 1.5 T, the use of FSE can require compromises. First, for FSE and all other sequences, the effective wavelength of the radio frequency (RF) magnetic field becomes a contributing factor to RF homogeneity degradation (1, 2). This degraded RF homogeneity will result in a distribution of refocus flip angles across the sample that will contribute to a decrease in image homogeneity. Second, increased power deposition has placed new boundaries on the use of radio frequency (RF) intensive sequences (3-5). At magnetic fields of 3.0 T and higher, reducing the power of the refocus RF pulses is often required to conform to RF heating guidelines (6-8). Spin echo sequences utilizing reduced refocus flip angles have been shown to provide suitable signal amplitudes and sensitivity (9, 10). However, the reduction of refocus pulse power also creates an unexpected penalty of further increasing signal variation across the image. At 1.5 T, wavelength effects do not have a significant effect and the loss of homogeneity is less noticeable. However at 3.0 T, the loss of homogeneity with reduced refocus pulse power is visibly evident. This effect is demonstrated both theoretically and experimentally with phantom and volunteer images at 1.5 T and 3.0 T.

[†]*A version of this chapter has been submitted for publication.*

5.1 Radio Frequency Homogeneity

5.1.1 Standing Waves

The electromagnetic wave theory presented in the previous section assumed that the electromagnetic wave was propagating through a uniform media. An electromagnetic wave that is obliquely incident on a boundary between two dielectric materials will have a component that is reflected back toward the first media and a component that is transmitted into the second media (11-13). If the boundary conditions allow free charge and free currents on a conducting surface, then the relations governing the transmission and reflection of an electromagnetic wave at a boundary are:

$$\epsilon_1 E_{1\perp} - \epsilon_2 E_{2\perp} = \sigma_f \quad [5.1a]$$

$$B_{1\perp} = B_{2\perp} \quad [5.1b]$$

$$\mathbf{E}_{1\parallel} = \mathbf{E}_{2\parallel} \quad [5.1c]$$

$$\frac{1}{\mu_1} \mathbf{B}_{1\parallel} - \frac{1}{\mu_2} \mathbf{B}_{2\parallel} = \mathbf{K}_f \times \hat{\mathbf{n}} \quad [5.1d]$$

where σ_f is the free surface charge, $\mathbf{K}_f$ is the free surface current at the boundary and $\hat{\mathbf{n}}$ is a unit vector that is perpendicular to the surface pointing from medium 2 into medium 1. An ohmic conductor requires there to be no free surface current ($\mathbf{K}_f = 0$) as this would require an infinite field at the boundary. In many NMR experiments the RF coil is situated outside the sample and the RF electromagnetic field will, at a minimum, need to cross from an air medium into the sample followed by a cross back into the air medium. During the first boundary crossing there will be some component of the RF field that is transmitted into the sample. Upon reaching the second boundary, some of this transmitted field will be transmitted out of the sample but a fraction of the field will be reflected back into the sample. When this reflected wave reaches the first boundary it will again have reflected and transmitted components (Fig 5.1).

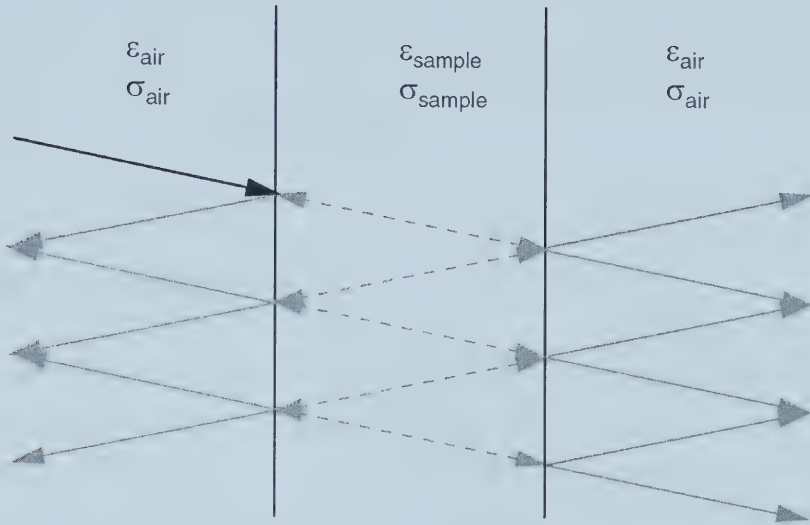


Figure 5.1: An incident electromagnetic wave will both reflect and transmit at each dielectric boundary it crosses. The reflected and transmitted waves in each media will both constructively and destructively interfere.

The reflected and transmitted waves within the sample will interact causing both destructive and constructive interference (14).

The resulting field pattern will depend critically on the electric properties of the sample as well as the frequency of the incident electromagnetic wave (2). The sample will act as a dielectric waveguide that will only support certain modes or standing waves. As the wavelength of the RF field approaches the dimensions of the sample, the standing wave effect will lead to an increasingly more inhomogeneous RF field (3). The wavelength in the sample is:

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon\mu}} \quad [5.2]$$

where λ_0 is the wavelength of the electromagnetic wave in a vacuum.

In general, it is the permittivity of the sample that will contribute the most to the shortening of the effective wavelength. The wavelength in free space of the RF field is also inversely dependent on magnetic field strength and this will have important consequences in high field NMR experiments (15, 16).

5.1.2 RF Field Mapping

The RF field within a sample during a RF pulse will not be spatially uniform and this will lead to undesirable intensity variations in MR images (17, 18). The homogeneity of the RF field will depend on the homogeneity of the RF coil as well as the electromagnetic field variations due to RF penetration (conductivity dependent) and standing wave (permittivity dependent) effects. Several workbench and MRI based methods have been proposed to map RF fields (2, 19-26). Only one of these methods utilizing spin echo imaging will be examined more closely (2, 26).

This spin echo method, used by Barker et al. at 1.5 T and Alecci et al. at 3.0 T, requires a derivation of signal dependence on flip angle proposed by Glover et al. (16). A spin with an offset frequency $\Delta\omega$ is excited with a θ_x RF pulse. Following the excitation, the spin will acquire a phase $\phi = \Delta\omega \cdot \tau$ during the time interval τ . The spin is then refocused with a $2\theta_y$ RF pulse and allowed to evolve for a second time interval τ . Neglecting relaxation, the magnetization during acquisition is:

$$M_x = M_0 [\sin(\theta) \cos^2(\theta) \sin(2\phi) - \cos(\theta) \sin(2\theta) \cos(\phi)] \quad [5.3a]$$

$$M_y = M_0 [\sin(\theta) \{ \cos^2(\phi) - \cos(2\theta) \sin^2(\phi) \} + \sin(2\theta) \cos(\theta) \sin(\phi)] \quad [5.3b]$$

An ensemble of spins with a uniform phase distribution will have a total transverse signal modulus of:

$$|M_{xy}| = \sqrt{\left(\frac{1}{2\pi} \int_{-\pi}^{\pi} M_x d\phi\right)^2 + \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} M_y d\phi\right)^2} \quad [5.4]$$

The x-component averages to zero and the final result is:

$$|M_{xy}| = M_0 \sin^3(\theta) \quad [5.5]$$

Using the signal dependence of Eq. [5.5], the resulting signal intensity when utilizing a transmit/receive birdcage coil is given by:

$$S(\mathbf{r}) = k \cdot D(\mathbf{r}) \cdot B_1(\mathbf{r}) \cdot \sin^3 \left[\frac{\pi}{2} \cdot \frac{p}{p_{\pi/2}} \cdot B_1(\mathbf{r}) \right] \quad [5.6]$$

where $S(\mathbf{r})$ is the spatially dependent signal intensity, $B_1(\mathbf{r})$ is a normalized RF field distribution in the sample, $D(\mathbf{r})$ represents the proton density and k is an arbitrary constant that depends on several system parameters such as the receiver gain. The transmit power corresponding to a 90° excitation flip angle (typically calibrated at the center of the RF coil) is denoted by $p_{\pi/2}$ and p is the actual transmit power used in the experiment. The field distribution $B_1(\mathbf{r})$ depends on the RF coil, RF penetration and field fluctuations due to the standing wave effect.

Any spatial signal variation observed in Eq. [5.6] is attributed to either spatial fluctuations in the proton density $D(\mathbf{r})$ or the RF field distribution $B_1(\mathbf{r})$. For the case of phantom data, the proton density is homogeneous across the sample and any signal intensity deviation in Eq. [5.6] is attributed to variations in $B_1(\mathbf{r})$.

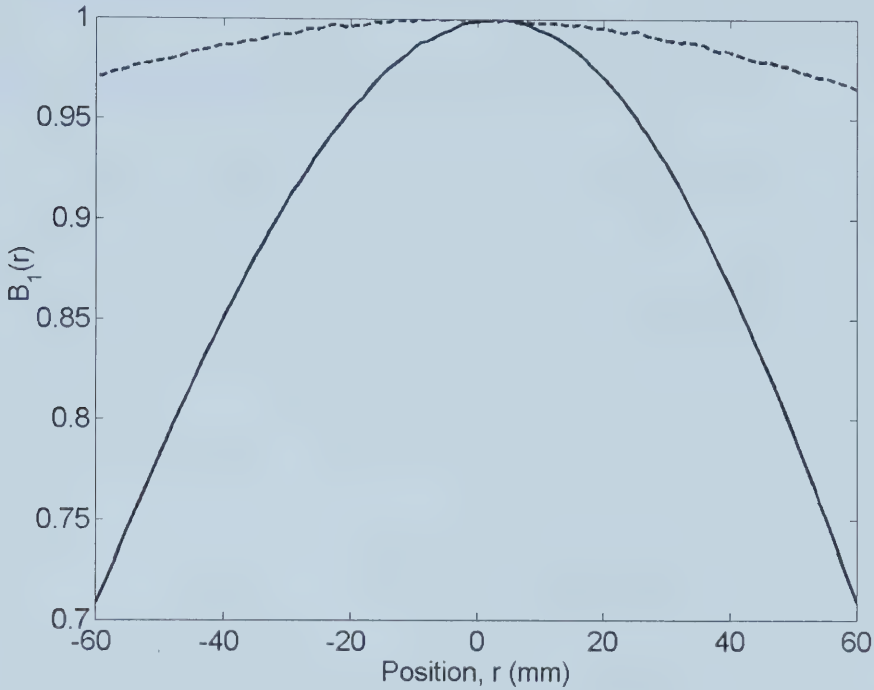


Figure 5.2: Plot of $B_1(r)$ for a phantom at 1.5 T (dashed line) and 3.0 T (solid line).

A spin echo phantom image acquired at both 1.5 T and 3.0 T has been used with Eq. [5.6] to extrapolate the RF field distributions for the phantom (Fig. 5.2). At 1.5 T, this particular phantom suffers from a 3% drop in $B_1(r)$ at the edges with respect to the center of the sample. In comparison, at 3.0 T there is a significantly less homogeneous RF field distribution with a drop of 29% at the phantom edges with respect to the center of the sample.

As Fig. 5.2 helps to illustrate, the general trend is that the RF field distribution will be less homogeneous as magnet field strength increases. The RF distribution is further complicated when a sample consists of multiple materials each with different electrical characteristics. Knowledge of the RF field distribution is essential in predicting how the flip angle and power deposition will vary spatially within a slice.

5.2 Reduced Refocus Flip Angles

5.2.1 Signal Intensity With Reduced Flip Angles

The $\sin^3\theta$ signal dependence of Eq. [5.6] was based on the assumption that the refocus RF pulse ($2\theta_y$) was twice that of the excitation pulse (θ_x) (16). To study the effects of the refocus pulse flip angle on signal intensity, a new equation with a refocus pulse power that is independent of the excitation pulse power must be found. Using the same method as Glover et al. in the derivation of Eq. [5.6], the magnetization of a spin undergoing a sequence with variable refocus pulse power (θ_x - τ - $\xi\theta_y$ - τ -acquire) is:

$$M_x = M_0 \left[\sin(\theta) \cos^2\left(\frac{\xi\theta}{2}\right) \sin(2\phi) - \cos(\theta) \sin(\xi\theta) \cos(\phi) \right] \quad [5.7a]$$

$$M_y = M_0 \left[\sin(\theta) \{ \cos^2(\phi) - \cos(\xi\theta) \sin^2(\phi) \} + \sin(\xi\theta) \cos(\theta) \sin(\phi) \right] \quad [5.7b]$$

An ensemble of spins with a uniform phase distribution will have a modulus of the total transverse signal of:

$$|M_{xy}| = \sqrt{\left(\frac{1}{2\pi} \int_{-\pi}^{\pi} M_x d\phi \right)^2 + \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} M_y d\phi \right)^2} \quad [5.8]$$

The signal intensity, $S(\mathbf{r}, \xi)$, of a sequence with variable refocus pulse power (θ_x - τ - $\xi\theta_y$ - τ -acquire) utilizing a transmit/receive birdcage coil is derived as:

$$S(\mathbf{r}, \xi) = k \cdot D(\mathbf{r}) \cdot B_1(\mathbf{r}) \cdot \sin\left[\frac{\pi}{2} \cdot \frac{p}{p_{\pi/2}} \cdot B_1(\mathbf{r})\right] \cdot \sin^2\left[\frac{\pi}{4} \cdot \frac{\xi p}{p_{\pi/2}} \cdot B_1(\mathbf{r})\right] \quad [5.9]$$

The constant ξ can be adjusted to control the power of the refocus pulse. Equation [5.9] reduces to Eq. [5.6] when the refocus flip angle is exactly twice the excitation flip angle ($\xi = 2$). At magnetic fields of 3.0 T and higher, reducing the power of the refocus RF pulses is often required to conform to RF heating guidelines (6-8). Spin echo sequences utilizing reduced refocus flip angles have been shown to provide suitable signal amplitudes and sensitivity (9, 10). Nevertheless, for FSE and all other sequences, the effective wavelength of the radio frequency magnetic field becomes a contributing factor to RF homogeneity degradation (1, 2). This degraded RF homogeneity will result in a distribution of refocus flip angles across the sample that will contribute to a decrease in image homogeneity as shown in Fig. 5.3.

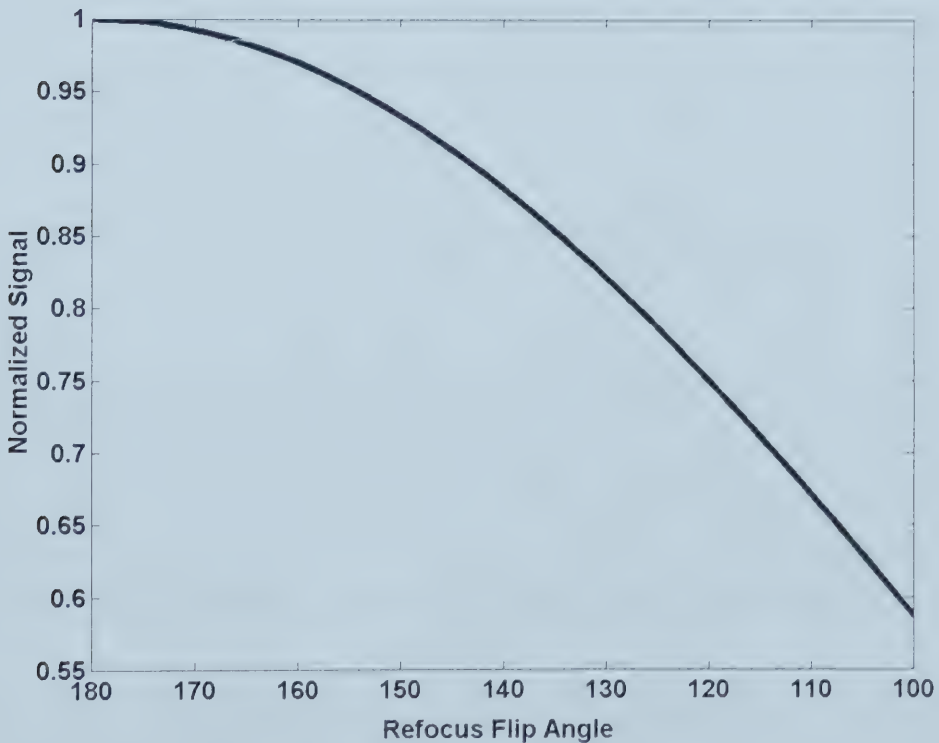


Figure 5.3: Normalized signal intensity for a spin echo sequence as a function of the refocus pulse flip angle.

5.2.2 Effect of Reduced Flip Angles on Image Homogeneity

The reduction of refocus pulse power creates an unexpected penalty of further increasing signal variation across an MR image. At 1.5 T, wavelength properties do not have a significant effect on RF homogeneity and the loss of signal homogeneity is less noticeable. However at 3.0 T, the loss of homogeneity with reduced refocus pulse power is visibly evident.

To demonstrate this, the excitation pulse of a $(\theta_x\text{-}\tau\text{-}\xi\theta_y\text{-}\tau\text{-}\text{acquire})$ spin echo sequence will be calibrated to give a 90° flip angle at the center of the coil/sample ($p = p_{\pi/2}$). Since signal variations resulting from a non-uniform proton density are not the focus, it is useful to normalize the signal intensity of a $B_1(\mathbf{r})$ distribution with the signal intensity that would be obtained in the absence of any RF degradation (i.e. $B_1(\mathbf{r}) = 1$), thus removing the influence of $D(\mathbf{r})$ and k in Eq. [5.9]. The resulting normalized signal is defined below.

$$S_N(\mathbf{r}, \xi) = \frac{S(\mathbf{r}, \xi)}{S(\mathbf{r}, \xi)_{B_1(\mathbf{r})=1}} = \frac{B_1(\mathbf{r}) \cdot \sin\left[\frac{1}{2}\pi B_1(\mathbf{r})\right] \cdot \sin^2\left[\frac{1}{4}\xi\pi B_1(\mathbf{r})\right]}{\sin^2\left[\frac{1}{4}\xi\pi\right]} \quad [5.10]$$

The function $S_N(\mathbf{r}, \xi)$ provides spatial measurements of signal variation that are a direct result of the degraded RF field distribution and reduced refocus pulse power. The primary interest is in the loss of image homogeneity with the reduction of refocus pulse flip angle. An equation that compares the signal variations of a reduced refocus pulse image to the signal variations of an image utilizing full power refocus pulses ($\xi = 2$) is:

$$H(\mathbf{r}) = \left| \frac{S_N(\mathbf{r}, 2) - S_N(\mathbf{r}, \xi)}{S_N(\mathbf{r}, 2)} \right| \times 100\% = \left| 1 - \frac{\sin^2\left[\frac{1}{4}\xi\pi B_1(\mathbf{r})\right]}{\sin^2\left[\frac{1}{2}\pi B_1(\mathbf{r})\right] \cdot \sin^2\left[\frac{1}{4}\xi\pi\right]} \right| \times 100\% \quad [5.11]$$

The ratio $H(\mathbf{r})$ gives the percentage increase in signal variation as a function of refocus pulse power and RF field distribution. Figure 5.4 demonstrates that as the RF field distribution becomes more inhomogeneous, the amount of signal variation $H(\mathbf{r})$ is expected to increase. Of more importance to this section is that the inhomogeneity increases more quickly with $B_1(\mathbf{r})$ for the reduced refocus flip angles. A consequence of this is that given a RF field distribution in a sample, the resulting spin echo image will have more signal variation and therefore be less homogeneous as the refocus flip angle is reduced. Although reduced refocus flip angles cause this effect for all degraded $B_1(\mathbf{r})$ values, the effect is more pronounced as $B_1(\mathbf{r})$ becomes less uniform.

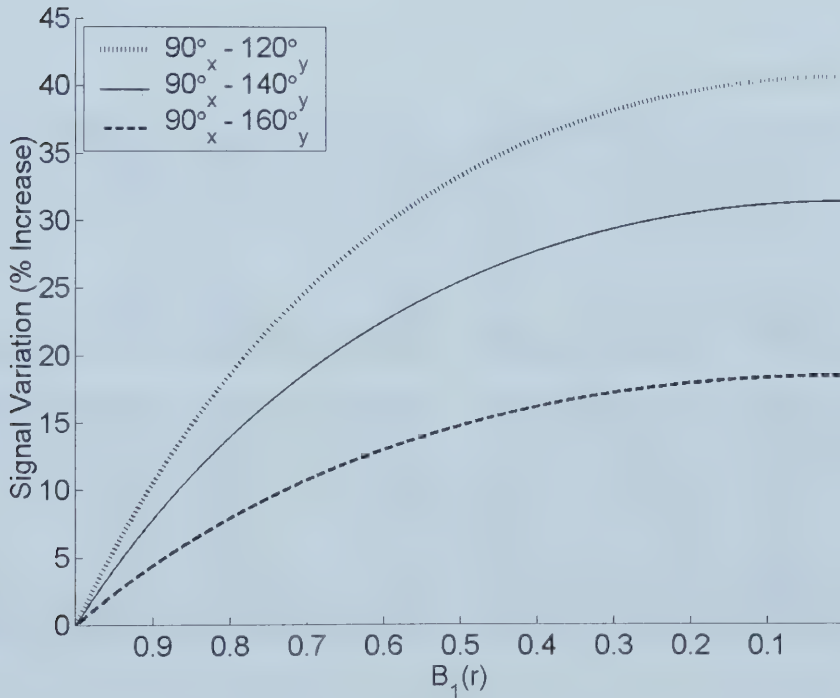


Figure 5.4: Graphs of the signal variation increase as a function of the $B_1(\mathbf{r})$ field distribution (Eq. [5.11]) for several different refocus pulse flip angles. The plots are labeled with the excitation and refocus flip angles when the RF field is not degraded (i.e. $B_1(\mathbf{r}) = 1$). Reduced refocus flip angles are seen to exhibit higher signal variation and will result in a less homogeneous image.

The signal variations in Fig. 5.4 make no assumptions about how $B_1(\mathbf{r})$ varies with magnetic field strength or position in the image. In order to correlate signal variations with position, the RF field distribution for the sample must be known. For the case of phantom data, the proton density is homogeneous across the sample and any signal intensity deviation in Eq. [5.9] is attributed to variations in $B_1(\mathbf{r})$. Spin echo and fast spin echo phantom images were acquired at both 3.0 T (SMIS console, Magnex magnet) and 1.5 T (Siemens Medical Systems Sonata). The cylindrical phantom had a diameter of 12 cm and consisted of 1.25 g $\text{NiSO}_4 \cdot 6\text{H}_2\text{O}$ + 5 g NaCl per 1000 g of distilled H_2O . At 1.5 T, the spin echo and fast spin echo (ETL=7) images used a 7.3 ms echo time and echo spacing. At 3.0 T, the spin echo and fast spin echo (ETL=8) images had a slightly longer echo time and echo spacing of 8 ms. A repetition time of 2000 ms was used for all the data sets.

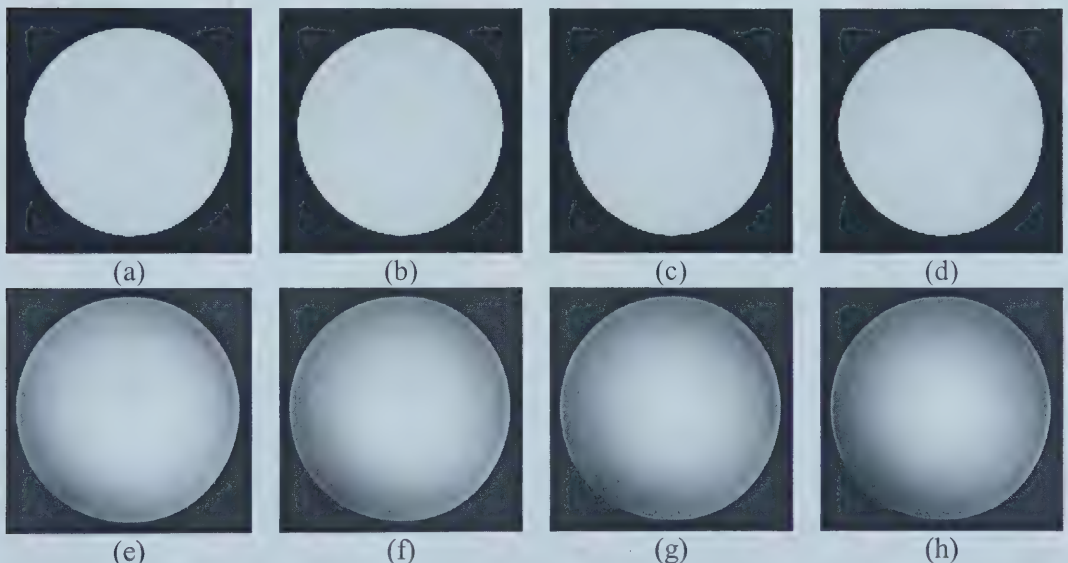


Figure 5.5: Phantom FSE images demonstrating the increased in-plane signal variation associated with reduced refocus pulse flip angles. The 1.5 T images (ETL = 7) have refocus flip angles of (a) 180°, (b) 160°, (c) 140° and (d) 120°. The 3.0 T images (ETL = 8) have the same refocus flip angles of (e) 180°, (f) 160°, (g) 140° and (h) 120°. The reduced inhomogeneity associated with lower refocus flip angles is not distinguishable at 1.5 T, but is visible at 3.0 T.

The $B_1(\mathbf{r})$ distribution of the phantom (Fig 5.2) is then used with Eq. [5.9] to predict the signal variation of reduced refocus pulse images acquired from the same phantom. Although spin echo data has been obtained at both field strengths, only the 3.0 T data has been presented graphically in Fig. 5.6.

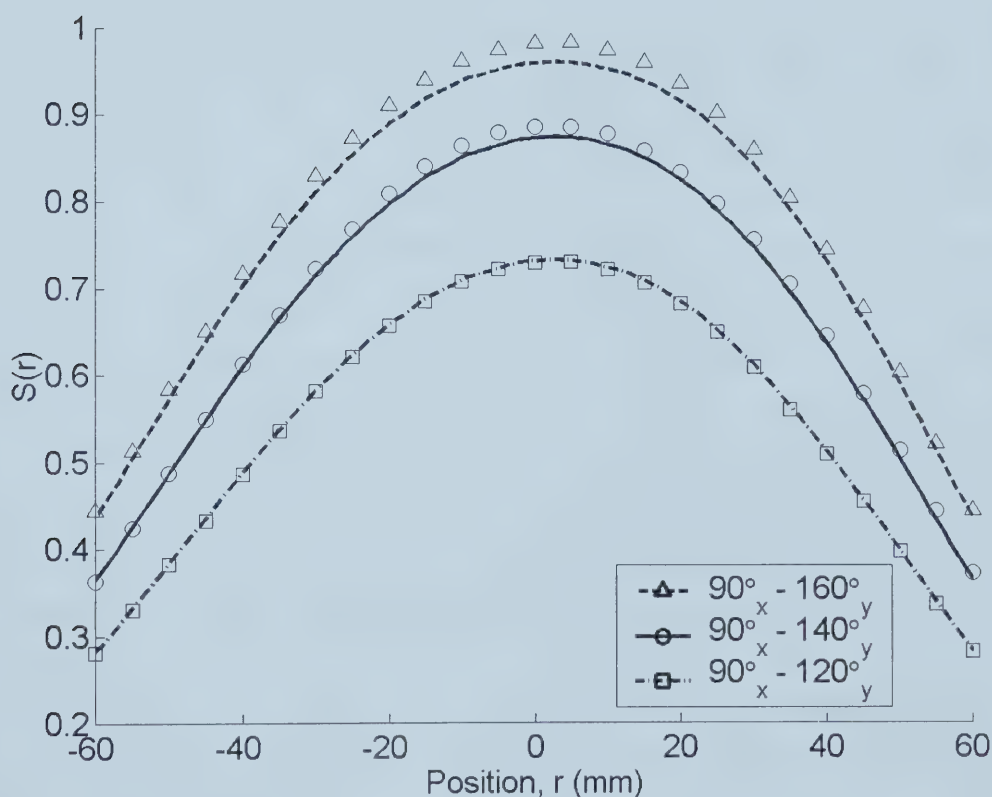


Figure 5.6: The plots show the signal variation for different refocus flip angles. The lines represent the theoretical curves, with the points being experimental data. The plots are spin echo phantom data with 160° (triangles with dashed line), 140° (circles with solid line) and 120° (squares with dashed/dotted line) refocus pulse flip angles at 3.0 T and are normalized to the maximum signal observed in the 180° refocus pulse image. The lower refocus flip angles not only suffer from an overall decrease in signal intensity, but the signal variation across the sample is also more significant.

At 1.5 T, the spin echo images have a 4% signal variation from center to edge when a 180° refocus pulse is used. Even when the refocus pulse is reduced to 120° , the amount of signal variation only increases by an additional 4%. In contrast, at 3.0 T the signal variation for a 180° refocus pulse spin echo image is 53% from the center to the edge of the phantom. When the refocus pulse flip angle is reduced to 120° , the 3.0 T image has a 24% increase to a 66% signal variation across the image.

For the spin echo equations, significant contributions to signal loss arise from the failure of the reduced refocus pulse to fully refocus any phase the spins acquired prior to the RF pulse. For fast spin echo sequences, the number of phase pathways becomes increasingly more complex and signal loss associated with not fully refocusing spins is not alleviated. The signal variation of the 1.5 T image, with 120° refocus pulses, increased 3% from an initial signal variation of 4% with 180° refocus pulse flip angles. The 3.0T fast spin echo images had a 26% increase in signal variation from the original 56% when the refocus pulse flip angles were reduced to 120° . Changing the echo time of the FSE images to a later echo along the train was found to have similar effects on image homogeneity. Table 5.1 summarizes that not only do the FSE phantom images suffer from approximately the same amount of base inhomogeneity, they suffer from approximately the same amount of increased inhomogeneity as refocus pulse flip angles are reduced.

In vivo images were also acquired at 1.5 T and 3.0 T from normal volunteers. For both image sets, a 512×512 matrix was acquired from a 5mm slice, with one average in a 22 cm FOV. The 1.5 T images were acquired with an echo train length (ETL) of 15 while the 3.0 T images had an ETL of 16. Both sets used a bandwidth of 50 kHz, an echo time and echo spacing of 10 ms and a 2000 ms repetition time. The signal intensity near the center of the image was measured in the thalamus (gray matter) while cortical gray matter surrounding the superior temporal and superior frontal gyrus was used to determine the signal intensity at the extremities of the image.

Table 5.1: Increased In-plane Signal Variation^a of Reduced Refocus Flip Angles.

Refocus Flip Angle	SE Phantom		FSE Phantom		FSE Head	
	1.5 T	3 T	1.5 T	3 T	1.5 T	3 T
160°	6 %	59 %	5 %	63 %	2 %	31 %
140°	7 %	63 %	6 %	67 %	2 %	32 %
120°	8 %	66 %	7 %	70 %	3 %	33 %

^aEquation [5.11]

To confirm this effect in vivo, volunteer head images were acquired at both field strengths. Figure 5.7 illustrates a set of reduced flip angle FSE images acquired at both 1.5 T and 3.0 T. From table 5.1, the 1.5 T head images (Fig. 5.7a-d) only suffered from a 2% increase in signal variation with 120° refocus pulses compared to the 1% signal variation seen with the 180° refocus pulse images. The 3.0 T FSE images (Fig. 5.7e-h) had a base signal variation of 29% when 180° refocus pulse flip angles were used. This signal variation across the image increased by 14% when the refocus pulse flip angles were then reduced to 120°.

The phantom used has electric properties that are appreciably different than those encountered in vivo. However, as the strength of the magnetic field increases, the RF field distribution inside a human head will become increasingly less uniform and this phantom data is able to forecast the type of results expected at higher magnetic field strengths. Despite the differences in the $B_1(\mathbf{r})$ field distribution, the same trend of increased image inhomogeneity with reduced refocus flip angles was verified in both phantom and volunteer images.

The importance of this trend is that reducing the refocus pulse flip angle contributes to increased signal variation across the image at all field strengths. However, at higher magnetic field strengths when the RF wavelength becomes more significant, this effect is exacerbated when reduced refocus flip angles are utilized. Thus, when considering reduced flip angles as a method of reducing power deposition at higher field strengths, one must also consider the potential for reduced image homogeneity.

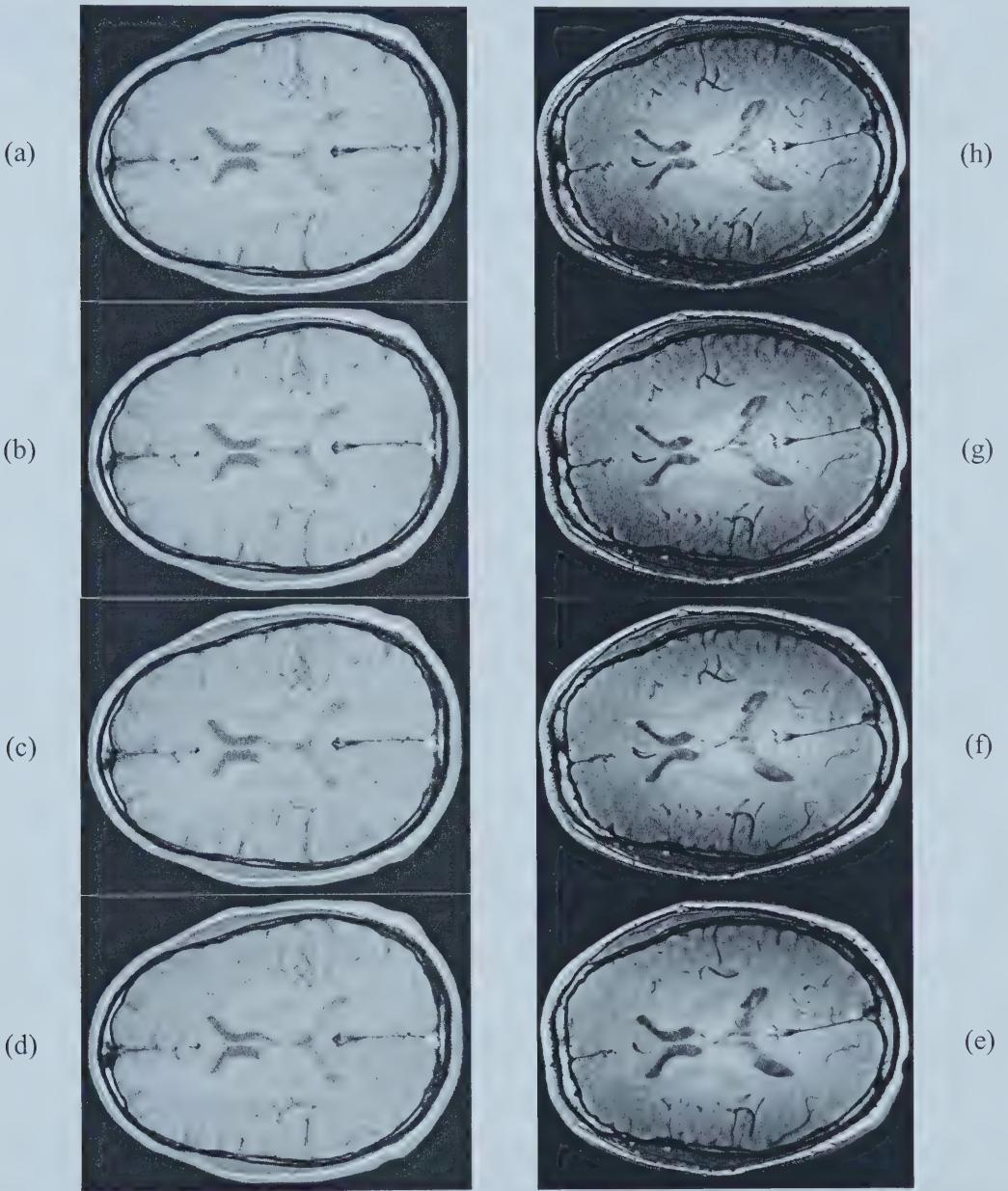


Figure 5.7: Volunteer head FSE images demonstrating the increased in-plane signal variation associated with reduced refocus pulse flip angles. The 1.5 T images (ETL = 15) have refocus flip angles of (a) 180°, (b) 160°, (c) 140° and (d) 120°. The 3.0 T images (ETL = 16) have the same refocus flip angles of (e) 180°, (f) 160°, (g) 140° and (h) 120°. The volunteer head images with lower refocus flip angles show little change in homogeneity at 1.5 T, but have an evident change at 3.0 T.

5.3 Bibliography

1. Vaughan JT, Garwood M, Collins CM, Liu W, DelaBarre L, Adriany G, Andersen P, Merkle H, Goebel R, Smith MB and Ugurbil K. 7T vs. 4T: RF Power, Homogeneity, and Signal-to-Noise Comparison in Head Images. *Magn Reson Med* 2001; 46: 24-30.
2. Alecci M, Collins CM, Smith MB and Jezzard P. Radio Frequency Magnetic Field Mapping of a 3 Tesla Birdcage Coil: Experimental and Theoretical Dependence on Sample Properties. *Magn Reson Med* 2001; 46: 379-385.
3. Hoult DI and Phil D. Sensitivity and Power Deposition in a High-Field Imaging Experiment. *J Magn Reson Imaging* 2000; 45: 46-67.
4. Hoult DI, Chen C-N and Sank VJ. The Field Dependence of NMR Imaging. *Magn Reson Med* 1986; 3: 730-746.
5. Collins CM and Smith MB. Signal-to-Noise Ratio and Absorbed Power as Functions of Main Magnetic Field Strength, and definition of "90°" RF Pulse for the Head in the Birdcage Coil. *Magn Reson Med* 2001; 45: 684-691.
6. Reynolds HG and Rydberg J. Average RF Power Reduction for T2 FLAIR at 3.0 Tesla. *Proceedings of the 8th Annual Meeting of ISMRM* 2000; 3: p1988.
7. Gati J, Thomas J, Steinman DA and Rutt BK. High Resolution Fast Spin Echo Black-Blood Imaging of Human Carotid Artery Wall at 4T. *Proceedings of the 9th Annual Meeting of ISMRM* 2001; p1964.

8. Guidance for Magnetic Resonance Diagnostic Devices. US Food and Drug Administration: Center for Devices and Radiological Health. Available from: <http://www.fda.gov/cdrh/ode/magdev.html>. Accessed February 2003.
9. Hennig J. Multiecho Imaging Sequences with Low Refocusing Flip Angles. *J Magn Res* 1988; 78: 397-407.
10. Alsop DC. The Sensitivity of Low Flip Angle RARE Imaging. *Magn Reson Med* 1997; 37: 176-184.
11. Griffiths DJ. Introduction to Electrodynamics. Upper Saddle River: Prentice-Hall Inc.; 1989. 532 pages.
12. Jackson JD. Classical Electrodynamics: 3rd Edition. New York: John Wiley & Sons Inc.; 1998. 808 pages.
13. Cheng DK. Field and Wave Electromagnetics. Reading: Addison-Wesley Publishing Company; 1992. 703 pages.
14. Marcuse D. Theory of Dielectric Waveguides. New York: Academic Press; 1974.
15. Bomsdorf H, Henzel T, Kunt D, Roeschmann P, Tschendel O and Wieland J. Spectroscopy and Imaging with a 4 Tesla Whole Body MR System. *NMR Biomed* 1988; 1: 151-158.
16. Glover GH, Hayes CE, Pelc NJ, Edelstein WA, Mueller OM, Hart HR, Hardy CJ, O'Donnell M and Barber WD. Comparison of Linear and Circular Polarization for Magnetic Resonance Imaging. *J Magn Reson* 1985; 64: 255-270.

17. Simmons A, Tofts PS, Barker GJ and Arridge SR. Sources of Intensity Non-uniformity in Spin Echo Images at 1.5 T. *Magn Reson Med* 1994; 32: 121-128.
18. Condon BR, Patterson J, Wyper D, Jenkins A and Hadley DM. Image Non-uniformity in Magnetic Resonance Imaging: Its Magnitude and Methods for Correction. *Br J Radiol* 1987; 60: 83-87.
19. Fakri-Bouchet L, Lapray C and Briguet A. Measurements of the Radio Frequency Field in Magnetic Resonance Coils. *Meas Sci Technol* 1998; 9: 1641-1646.
20. Murphy-Boesch J, So GJ and James TL. Precision Mapping of the B1 Field Using the Rotating Frame Experiment. *J Magn Reson* 1987; 73: 293-303.
21. Hornak JP, Szumowski J and Bryant RG. Magnetic Field Mapping. *Magn Reson Med* 1988; 6: 158-163.
22. Insko EK and Bolinger L. Mapping of the Radio Frequency Field. *J Magn Reson* 1993; 103: 82-85.
23. Jerschow A and Bodenhausen G. Mapping the B1 Field Distribution with Non Ideal Gradients in a High-Resolution NMR Spectrometer. *J Magn Reson* 1999; 137: 108-115.
24. Topp S, Adalsteinsson E and Spielman DM. Fast Multislice B1 Mapping. *Proc ISMRM* 1997; 281.
25. Sled JS and Bruce G. Standing Wave and RF Penetration Artifacts Caused by Elliptical Geometry: An Electrodynamics Analysis of MRI. *IEEE Trans Med Imaging* 1998; 17: 653-662.

26. Barker GJ, Simmons, Arridge SR and Tofts PS. A Simple Method for Investigating the Effects of Non-uniformity of Radiofrequency Transmission and Radiofrequency Reception in MRI. *Br J Radiol* 1998; 71: 59-67.

6

Conclusion

The focus of this thesis was to design, implement and evaluate the use of a fast spin echo imaging sequence on a 3.0 T MR scanner. In addition to accomplishing this, this research has extended the known theory on some of the essential relationships between refocus pulse flip angles, RF wavelength effects, and image homogeneity. Investigations by the author into various other aspects of the sequence design have resulted in the creation of a robust and stable FSE sequence. The FSE sequence is successfully programmed and currently operational on the 3.0 T MR scanner.

It has been demonstrated that the FSE sequence is a powerful imaging tool that can find applications in both clinical and research environments. Despite the increased power deposition at 3.0 T, FSE has been effectively applied beyond the head into the neck and even into whole body imaging. The several contrast mechanisms demonstrated make the FSE sequence an invaluable tool in clinical diagnostic imaging. Furthermore, the ability to include several features, such as fat saturation and magnetization transfer, into a single sequence makes the fast spin echo sequence extremely flexible in its functionality.

An investigation into the reduction of flip angles to overcome RF heating problems was carried out. The reduction of refocus pulse power created an unexpected penalty of further increasing signal variation across the image. At 1.5 T, wavelength effects did not have a significant effect on RF homogeneity and so the loss of image homogeneity was not noticeable. However at 3.0 T, the loss of homogeneity with reduced refocus pulse power was visibly evident. It was concluded that reducing the refocus pulse flip angle contributes to increased signal variation across the image at all field strengths. Furthermore, at higher magnetic field strengths when the RF wavelength becomes more significant, this effect is exacerbated when reduced refocus flip angles are utilized. Consequently the potential for reduced image homogeneity must be balanced with the need reduced flip angles as a method of reducing power deposition at higher field strengths.

Fast spin echo imaging was also shown to be successful in black blood Angiography. Several gradient designs were analyzed theoretically and tested experimentally. Black blood images obtained with the FSE sequence were presented. The benefits and failings of black blood imaging were discussed.

6.1 Future Directions

Several topics in the area of black blood Angiography require more research. The first is in the optimization of sequence parameters at higher field strengths. These include optimal echo times, refocus pulse flip angles and the phase of the refocus pulses. In the area of 3D FSE black blood imaging, problems associated with the creation of minimum intensity projections need to be overcome. First, structures such as bone and air pose a problem for minimum intensity projections since they appear dark in the black blood images. Any vessel information located above or below these structures is lost in the projection. A possible solution to this problem is in image segmentation to create local area projections. A similar problem arises at higher field strengths where RF field degradation results in a signal variation across the image, notably the loss of signal near the edges. Unfortunately this loss of signal is not confined to a structure or local area and cannot be removed through segmentation. Finally, further studies are needed to determine if black blood imaging is a clinically viable alternative to detecting vascular disease.

It also remains to be shown if the use of magnetization transfer can play a role in increasing the diagnostic capabilities of black blood imaging. It is not clear whether MT can successfully be used to help assess artifacts in black blood imaging caused by blood that has not been fully dephased.

More time should be devoted to the investigation of the role that hyperechoes can play in black blood imaging. Most particularly, in utilizing the concepts of hyperechoes to further dephase flowing spins while maintaining background tissue signal levels. The implementation of hyperecho sequences on the 3.0 T MR system will require a more precise method of controlling the phase of the RF pulses.

In the area of RF homogeneity, a further study of the effect of gradient design on the image signal variation is required. Preliminary data suggests that gradient designs that rewind phase before refocus pulses are more successful in preserving image homogeneity. It is also not certain whether surface coils, which have a sensitivity falloff opposite to the standing wave effect, might be utilized in helping to reduce the RF field distributions.

As the routine use of Magnetic Resonance Imaging moves to higher and higher field strengths, Fast Spin Echo sequences can continue to be utilized if research continues to find innovative solutions to the obstacles, such as increased power deposition and RF inhomogeneity, of FSE.

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